MHD Turbulence and large scale structures in the solar and heliospheric context

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This lecture will deal with how MHD turbulence coexists with some linear effects relevant for solar/heliospheric physics
We will consider mainly incompressible MHD, except in the solar wind case

PLAN
(1) Propagation and turbulence (homogeneous MHD with mean field)
(2) Stratification and turbulence (coronal heating)
(3) Expansion and turbulence (solar wind heating/anisotropy/turbulence))

LPP l'Observatoire de Paris LUTH
Laboratoire de l'Univers et de ses Théories

Les Houches 2011

Chapter 1: propagation and turbulence

Elsässer variables : $z^+ = u - b/\sqrt{\rho}$, $z- = u + b/\sqrt{\rho}$

MHD incompressible equations (simplified, 0 pressure) with mean field $B°\hat{e}_x$:

$\partial_t z^+ - B°\partial_x z^+ + NL^+ = 1/R\ \Delta z^+$ with $NL^+ = (z^-.\nabla)z^+ + \nabla P \approx \{(z^-.\nabla)z^+\} = z^+/\tau^+$

$\partial_t z^- + B°\partial_x z^- + NL^- = 1/R\ \Delta z^-$ with $NL^- = (z^+.\nabla)z^- + \nabla P \approx \{(z^+.\nabla)z^-\} = z^-/\tau^-$

$\text{div} z^{\pm} = 0$

Nonlinear times: $\tau^+ = 1/(kz^-)$, $\tau^- = 1/(kz^+)$

Linear propagation time: $\tau_A = 1/(\mathbf{k}.\mathbf{B°}) = 1/(k_{//}\ B°)$

Time ratio: $\mathbf{X = \tau_A/\tau^{\pm}}$

Anisotropy comes from X depending on wavenumber k AND angle (k,B°):

$X >> 1$ in *perp* directions ($\mathbf{k}.\mathbf{B}° \approx 0$) => NL wins

$X << 1$ in *parallel* directions ($\mathbf{k}.\mathbf{B}° \approx kB°$) => propagation wins

Anisotropy in Fourier space

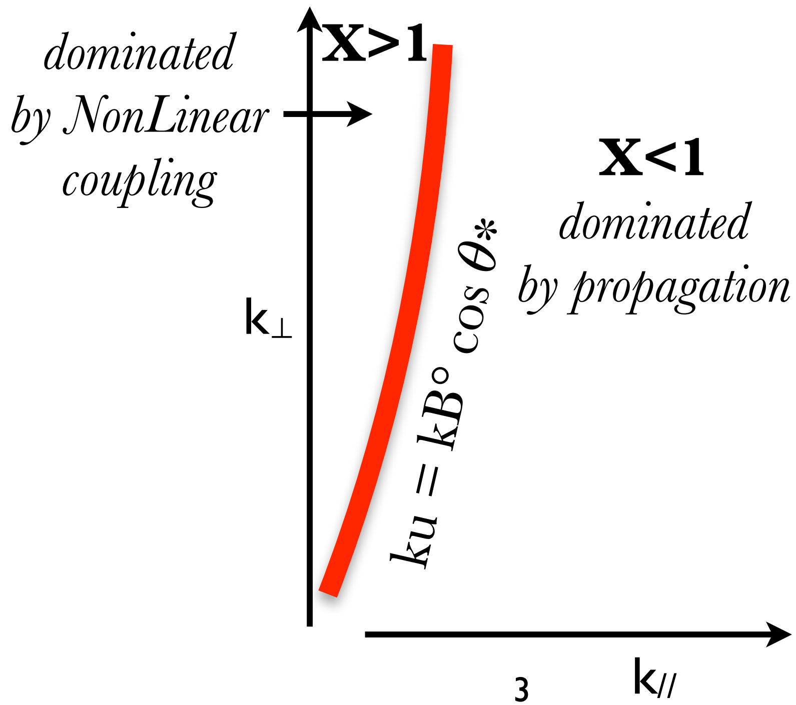
***Boundary** between NL and propagation dominated defined by* $\tau_{NL} = \tau_A$
Assuming $u \approx l^{1/3} \approx k^{-1/3}$:

$$\cos\theta^* \approx k^{-1/3}$$

Going to small scales (large k):
=> $\theta^* \rightarrow \pi/2$
=>Almost all modes dominated by propagation

The two regions

$X=\tau_A/\tau_{NL}$



Anisotropy in Fourier space: delayed IK cascade

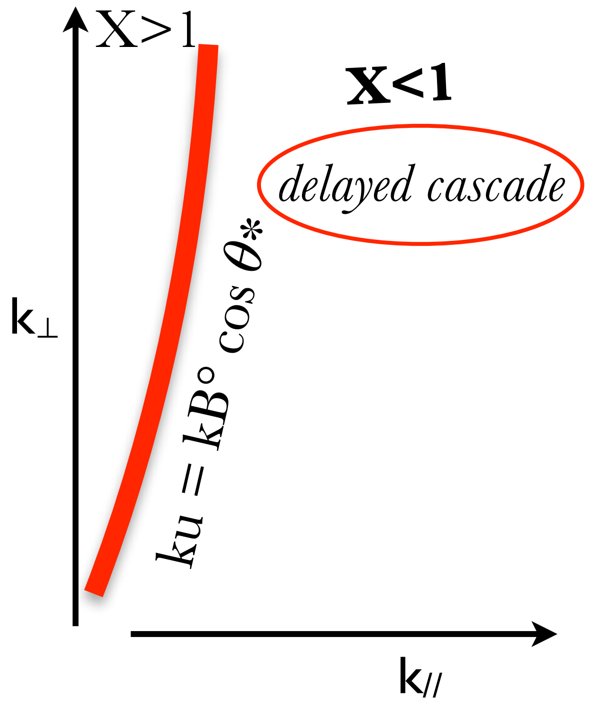
(Iroshnikov 1963, Kraichnan 1965)

•*Isotropize* region X<1: $\tau_A \approx \tau_A(\theta=0)$
•Assume incoherent interactions due to linear decorrelation of Alfvén wave packets (take into account NL ≈ $z^- z^+$)
=> **Delayed transfer time** $\tau^* \approx \tau_{NL}^2/\tau_A(\theta=0) >> \tau_{NL}$
•*Kolmogorov hypothesis of constant flux in k-space* => $k^{-3/2}$ spectrum

$k^{-3/2}$ spectrum found
- in 2D MHD with no mean field
- in 3D MHD with mean field (but see recent criticism by eg Lazarian)

Basic criticism:
- assumes isotropy in X<1 region: correct?
- ignore X>1 region: correct?

$X=\tau_A/\tau_{NL}$



Anisotropy in Fourier space: critical balance

(Goldreich Sridhat 1995)

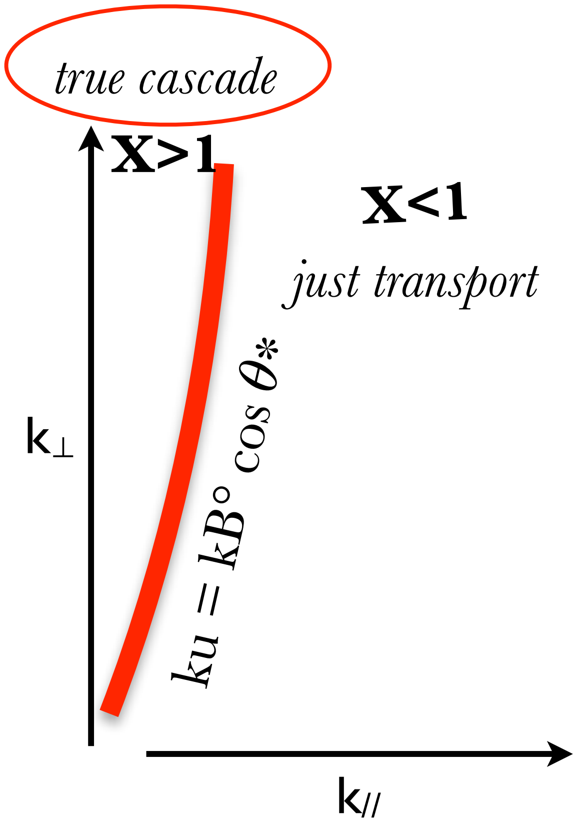
- Assume à la Kolmogorov cascade along $k_\perp$
- Assume **zero** cascade along $k_{//}$ (cf. Strauss equ., *reduced MHD*)
- Assume // spectrum generated passively by Alfvén transport such as (*critical balance*)

$$\tau_{//}^{cor} = \tau_\perp^{cor}$$

- Assume $\tau_\perp^{cor}=(k_\perp u)^{-1}$ and $\tau_{//}^{cor} = 1/k_{//}B°$

$k^{-5/3}$ spectrum found
- in 3D MHD with no mean field
- in 3D ideal MHD with mean field (see below)

Basic criticism
- no prediction of spectrum in different directions (only perp and //)
- reduced MHD assumption correct?



$X=\tau_A/\tau_{NL}$

Reduced MHD: check for critical balance

Critical balance is the first model attempting to describe the anisotropic MHD cascade, i.e. trying to relate parallel and perp. spectra. But is it true?

Defining Shell Model for reduced MHD

1) **Reduced MHD** <=> **ZERO parallel NL terms**

$\partial_t z^\pm$ -+ $B°\partial_x z^\pm + NL^\pm = 1/R\ \Delta z^\pm$ but with $\partial/\partial x \rightarrow 0$ in $NL^\pm$

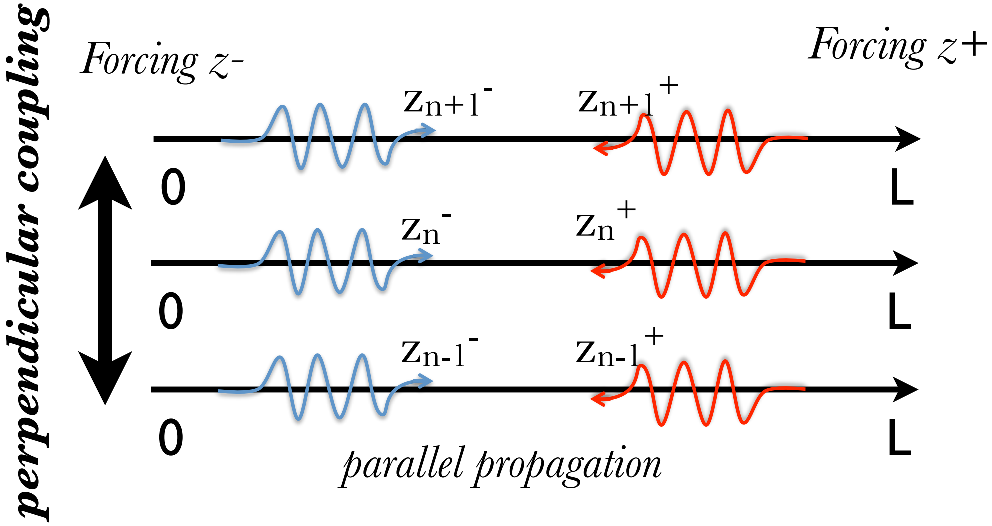
=> critical balance ***should work*** !

2) **SHELL model**: Replace perp NL terms by shell model (so, as many shell models as x grid points...)

NB Shell-Atm model allows reaching much larger Reynolds than reduced MHD because of logarithmic distretization of wave numbers

Buchlin & Velli. ApJ (2007), Nigro et al PRL (2004)

Shell model for reduced MHD with open boundaries

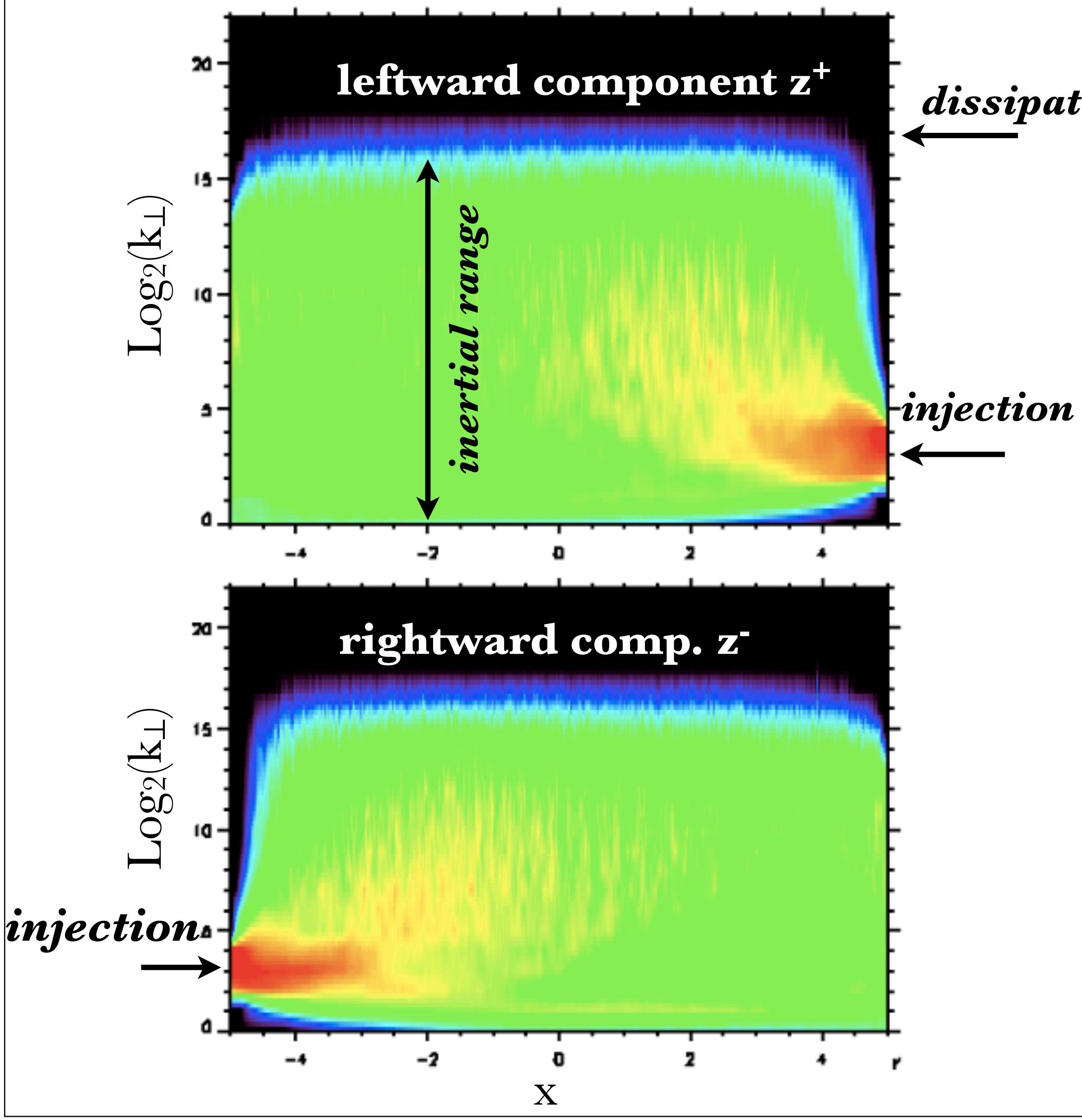


Shell-Atm Model:
- ***inject waves at boundaries***
- ***open boundaries*** : no reflexions of waves

$\partial_t z_n^+ - B^\circ \partial_x z_n^+ + NL_n^+ = f^+(n)$ if $x=L$
$\partial_t z_n^- + B^\circ \partial_x z_n^- + NL_n^- = f^-(n)$ if $x=0$
$n=1..N$ is number of perpendicular wavevector:
$k_n = k^\circ 2^n$

Spectra $E^{\pm}(x,k_\perp)$ (time-averaged)

Average spectra $k^{5/3}\ E^{\pm}(x,k)$

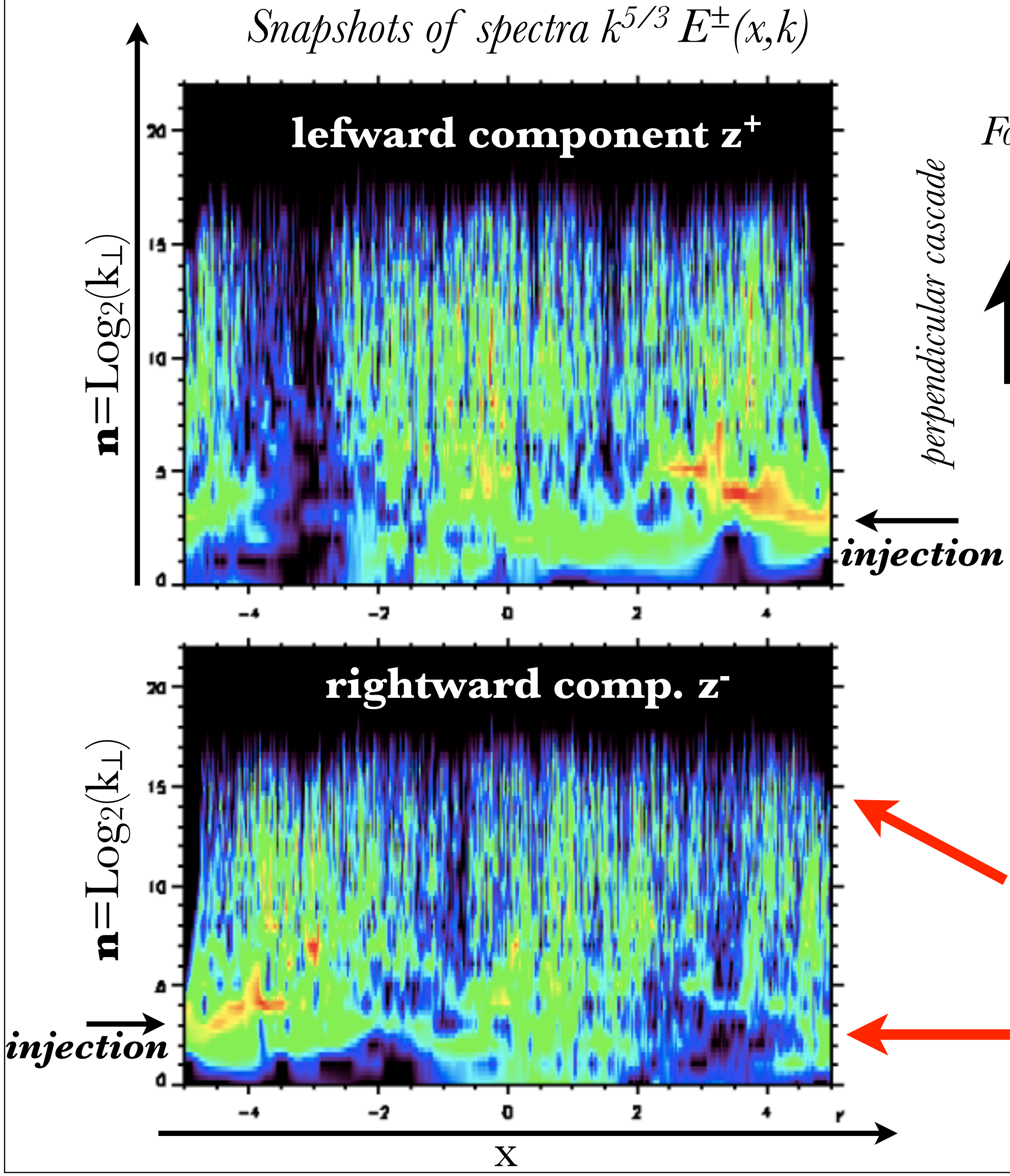


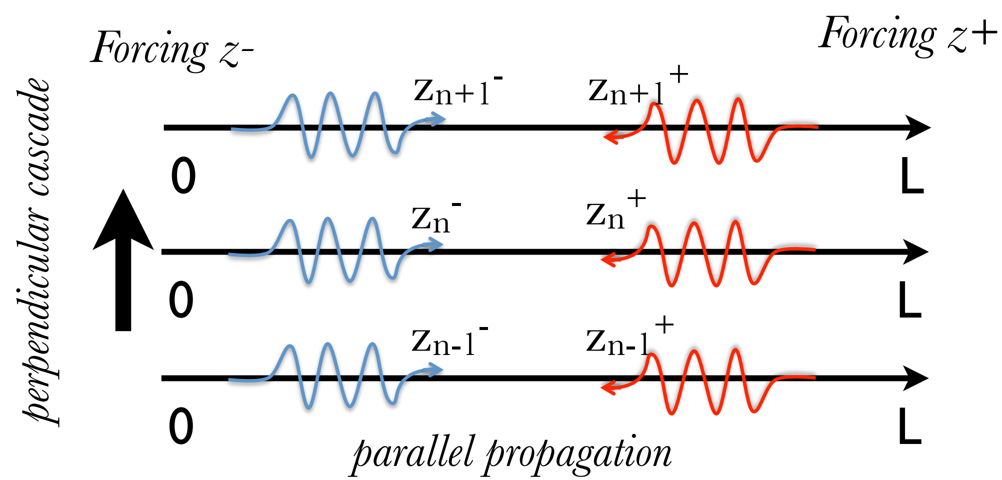
Perp. modes k_n injected at x=0 and L first propagate within the [0,L] domain. When z^+ and z^- modes meet, they start a **cascade** to smaller ⊥ scales (larger k_n). They form a spectrum $E^{\pm}(k,x)$, <u>different at each point x</u> (next slide)

<u>Here: time-averaged spectra</u>
(7 Alfvén times $t_A = L/V_a$)
show Kolmogorov scaling everywhere, except close to forced boundaries where forcing scales are enhanced

Note here forcing such that
$t_{NL}^{forcing} << t_A = L/Va$
=>spectra have time to develop during propagation

Snapshots of Z± spectra





Snapshots show **parallel structures,** which are smaller at smaller ⊥ scales

=> **is critical balance at work ?**

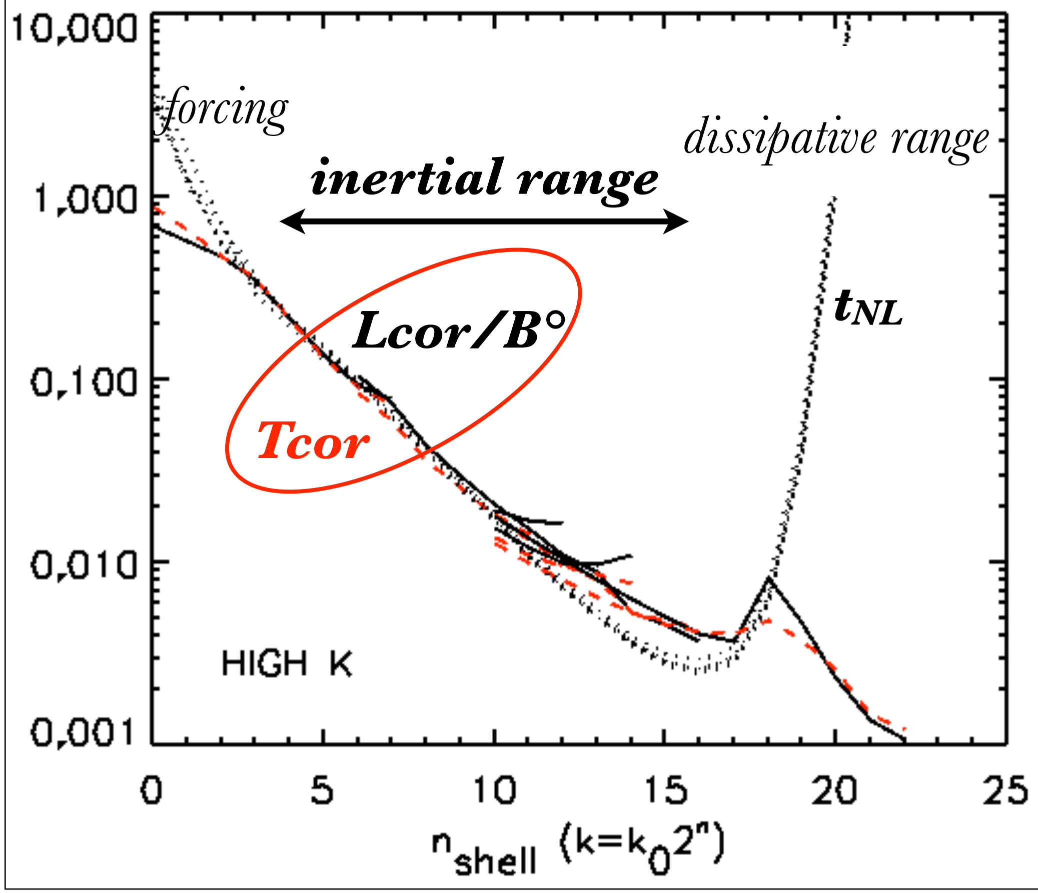
small // structures at large $k_\perp$

large // structures at small $k_\perp$

Critical balance within shell model ?

$\partial_t z_n^+ - B°\partial_x z_n^+ + NL_n^+ = f^+(n)$ at $x=L$
$\partial_t z_n^- + B°\partial_x z_n^- + NL_n^- = f^-(n)$ at $x=0$
$n=1..N$ is index of perpendicular wavevector: $k_n = k°2^n$
Boundaries are OPEN, $f(n)\neq 0$ for $n=1,2$

Tcor, Lcor/B°, t_{NL}



For each n, compute characteristic times ($n=\log_2(k)$):

1. Perp. correlation time $T_{cor}(n)$
2. // correlation length $L_{cor}(n)$ and // correlation time $L_{cor}/B°$
3. NL time $t_{NL}(n) = (k_n|u_n|)^{-1}$

In inertial range, all three time scales coincide:
Shell model verify critical balance

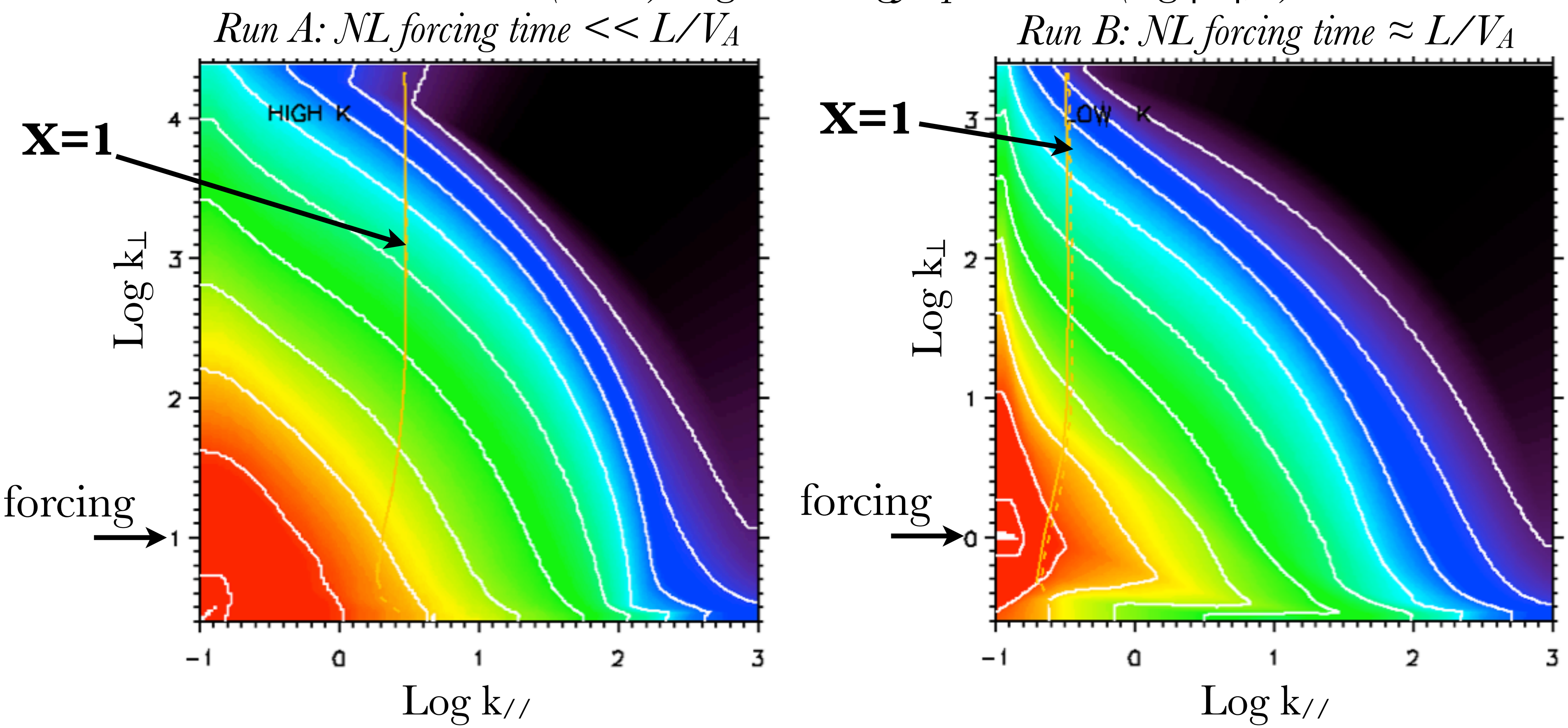
Full angular spectra $E(k_{//},k_{\perp})$: anisotropy

Spectra $E^{\pm}(x,k)$ can be **transformed into *full spectra*** $E\pm(k_{//}, k_{\perp})$ by taking FFT along x axis

NB Using cartesian-Log coordinates spoils anisotropy in Fourier space

=> $\mathbf{E^{\pm}(\log|k|)}$ in each direction θ should be used instead, as below

(Polar) angular energy spectrum $E(\log|k|,\theta)$



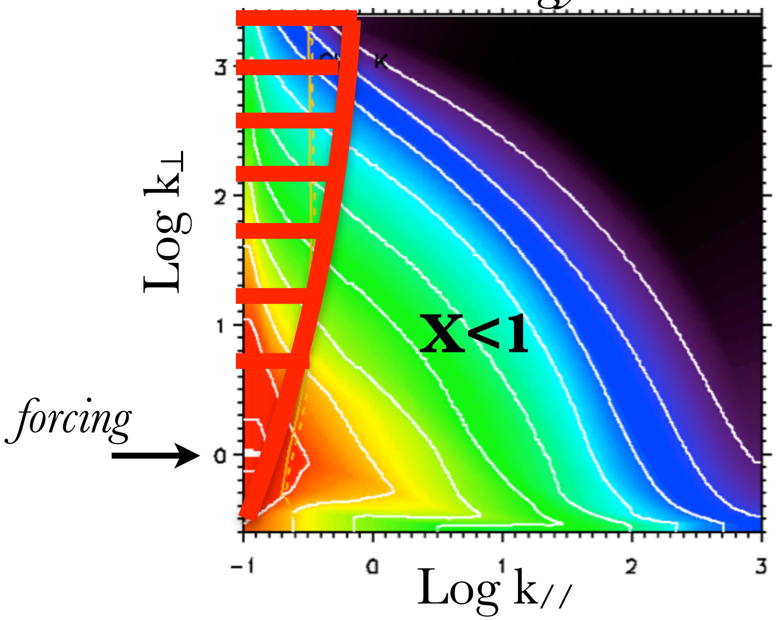
Comparing angular spectra

1. Shell model (forced by boundaries)
- Scaling in perp direction ≈ $\mathbf{k}_\perp^{-5/3}$
- but true only on $k_\perp$ axis (largest // scale)
- **Contrast** with, **in red**, contours of *critical balance spectrum* (*Goldreich Sridhar 1995*):
$E(k_\perp, k_{//}) = \mathbf{k}_\perp^{-5/3} f(X)$
$X>1 \Rightarrow f \approx 1$
$X<1 \Rightarrow f \approx 0$

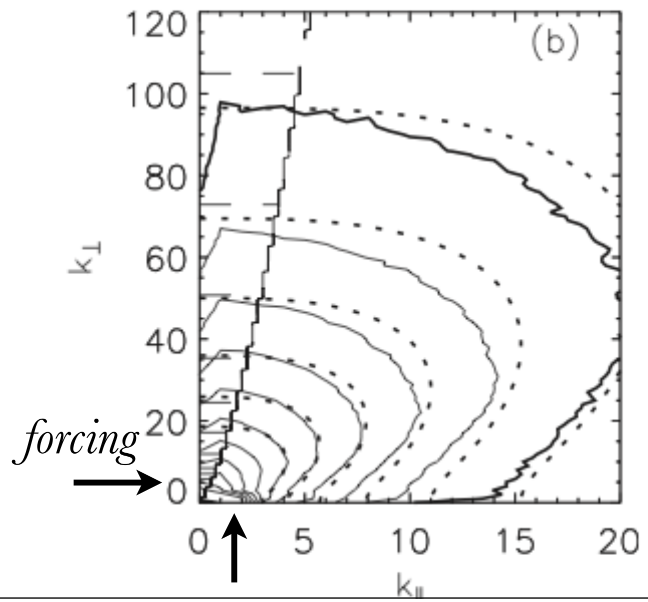
2. Full 3D MHD (forced in volume)
- *Same* scaling in all directions (anisotropy in amplitude only)
- Enhanced oblique directions compared to shell model

Grappin Müller PRevE 2010

Shell model energy contours



Full MHD energy contours



Conclusion Part 1

Strong part of critical balance verified in Shell model for reduced MHD
Weak part (spectral anisotropy) NOT verified

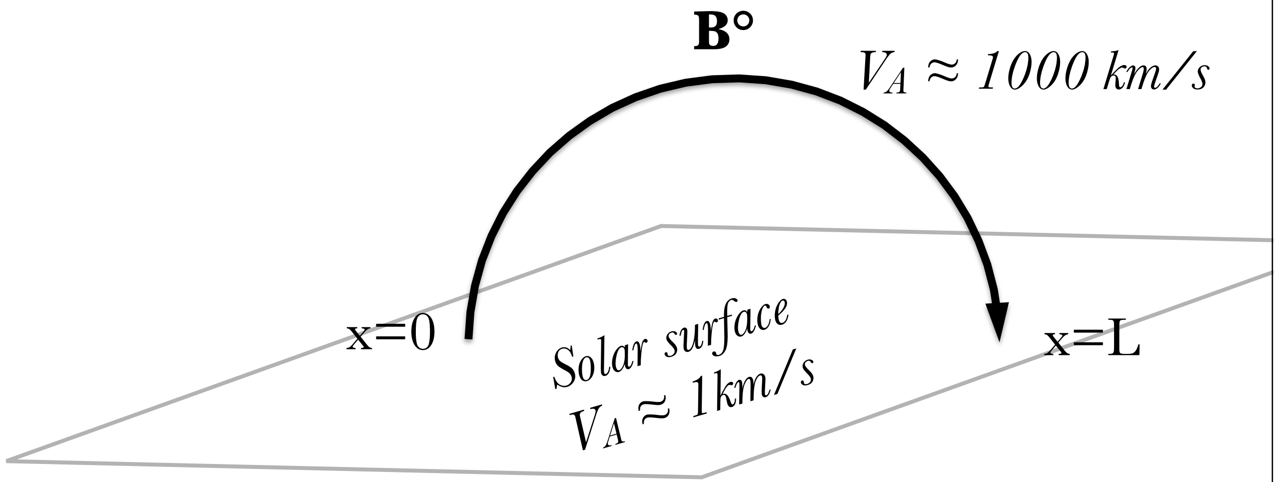
Note reduced MHD paradigm also can be criticized

Chapter 2: photosphere-corona coupling:
the closed-boundary or "line-tied" approximation

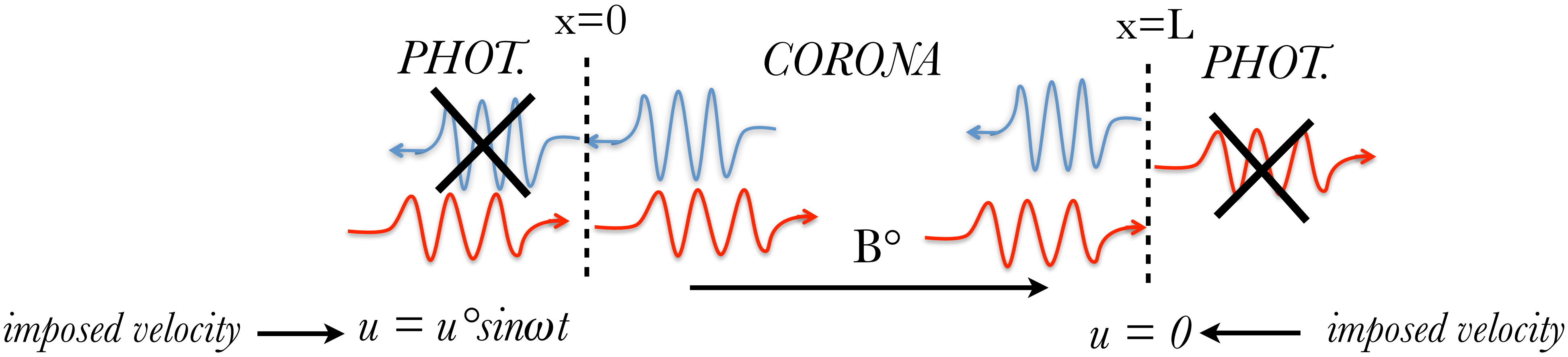
Closing boundaries of Shell model of reduced MHD
=> ***model for coronal heating***. Why?

Because corona = hot rarefied region above cold dense layers
=> Alfvén speed jump (≈100)
=> Boundary allowing
- perfect transmission of movements from photosphere
- perfectly reflecting waves once trapped in the corona
= "closed boundary", i.e. imposing velocity (here velocity perp. spectrum)

NB True for low frequency waves



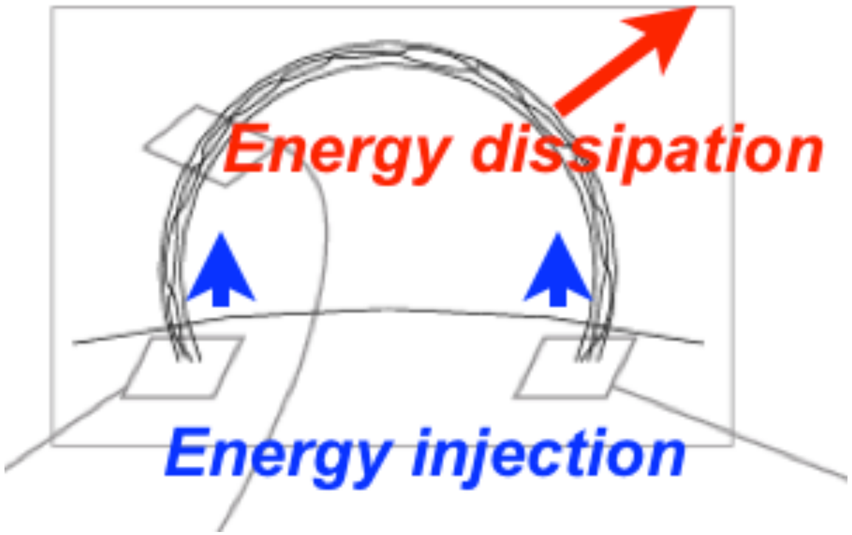
Closing boundaries ("line-tied")



•Movements are imposed at boundaries, where Alfvén waves are reflected
•Magnetic energy can increase and accumulate when frequency is *resonant (or zero)*
$E_b \approx E_v/(1-\cos 2\omega t_A)$
(t_A = crossing Alfvén time)

=> *only <u>turbulent dissipation</u> can balance photospheric boundary forcing*

Examples of
coronal dissipation balancing photospheric forcing



(a) **Reduced MHD** (B° large) *Rappazzo et al 2007*

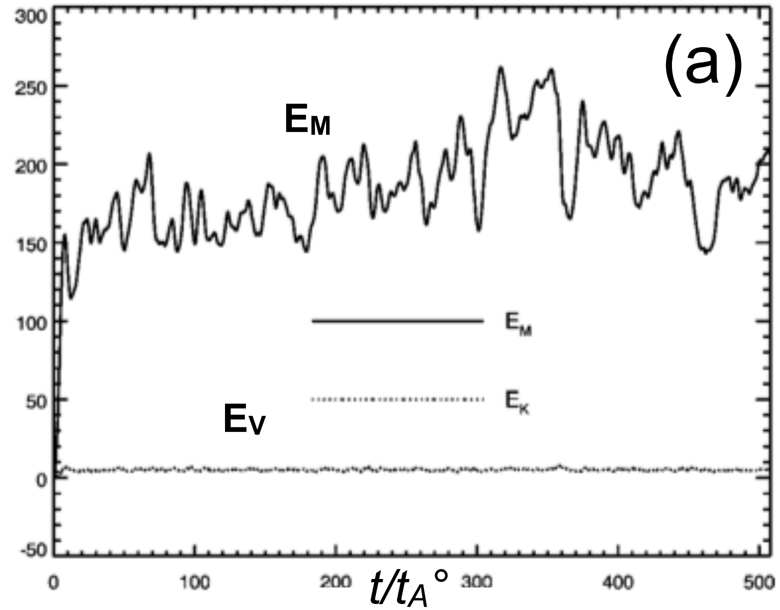
=> weak turbulence vs t_A/t_{NL}

(b) **Shell-Atm model**

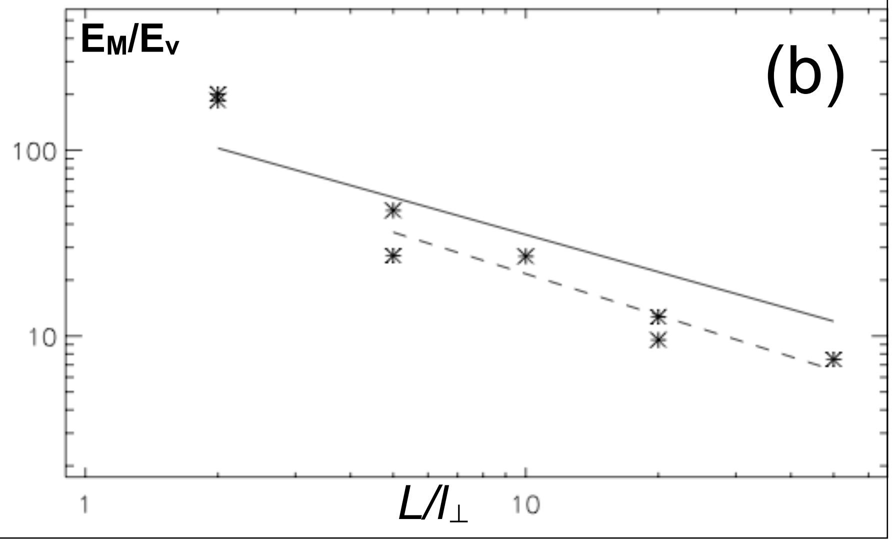
Buchlin & Velli. ApJ (2007), Nigro et al PRL (2004)

=> $(E_M/E_V)_{corona} = f(L/l_\perp)$

Coronal energies vs time



Coronal B/V ratio vs aspect ratio



Second source of energy loss: leakage

Finite Alfvén speed jump between photosphere and corona
=> boundaries are NOT perfectly reflecting: energy "leaks"
Leakage time is a long time:

$$t_L = L/v_A^{phot} \approx 100\ L/v_A^{corona}$$

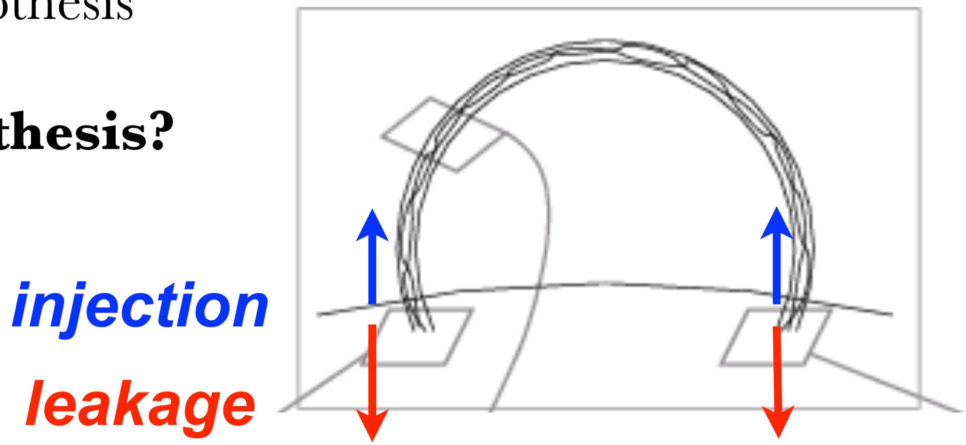
(Hollweg 1984, Ofman 2002, Grappin Aulanier Pinto 2008)

Hollweg assumed **leakage always longer than the dissipative** time:

$$t_L >> t_{NL}$$

which led people to adopt the line-tied hypothesis

=> What about **testing Hollweg's hypothesis?**



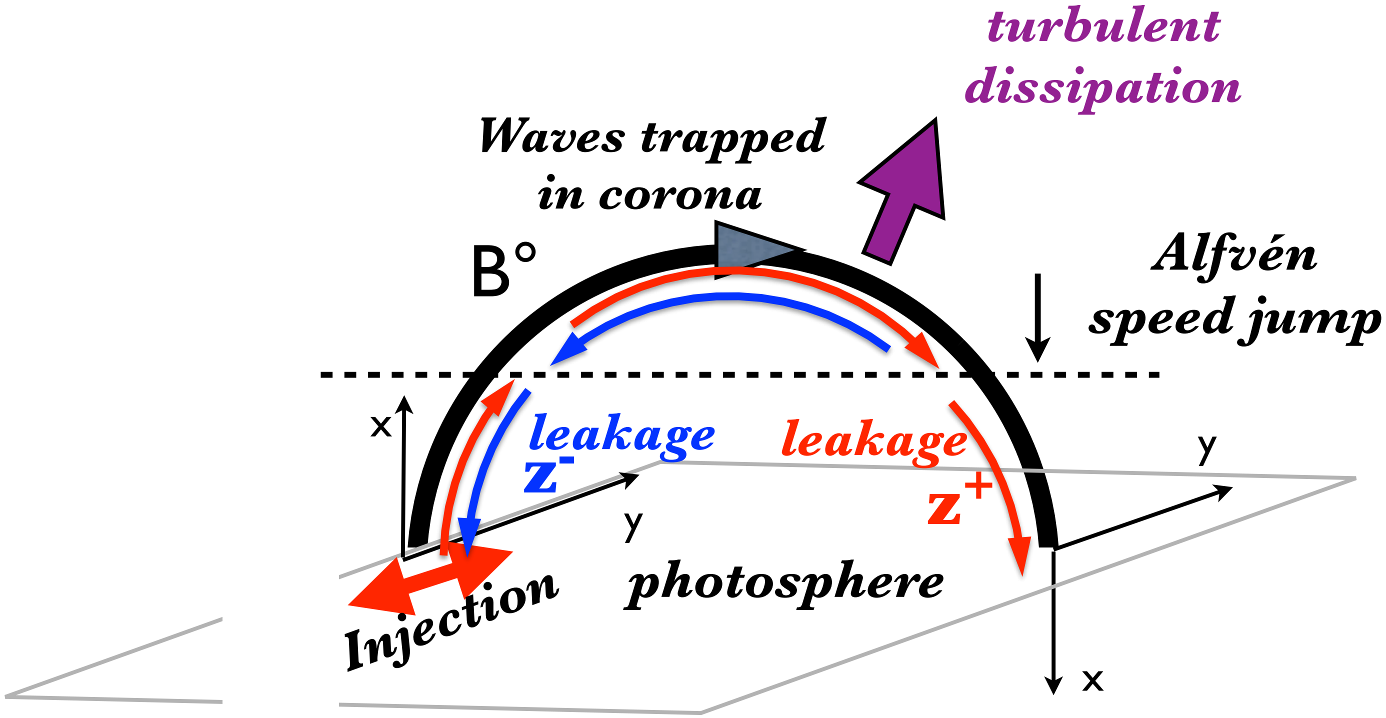
Leakage AND turbulence
SHELL model with chromosphere+corona

•Take Shell-Atm model, use *open* boundaries at x=0 and L (solar surface)

•Include a "boundary" within the domain a *finite Alfvén speed jump*:

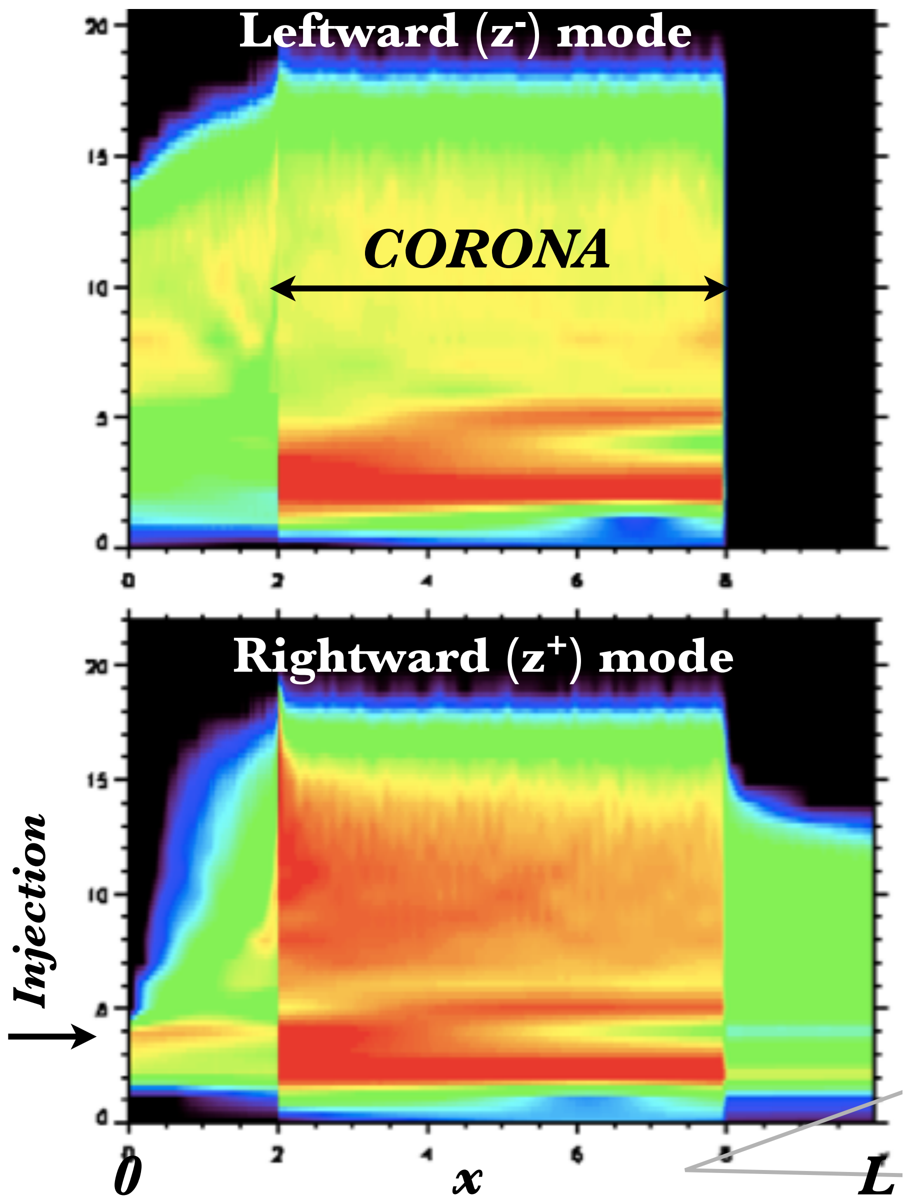
Va=100 km/s in corona, Va=1 km/s in photosphere (say)

•Inject waves at *left* foot point x=0.



Leakage AND turbulence
SHELL model with chromosphere+corona

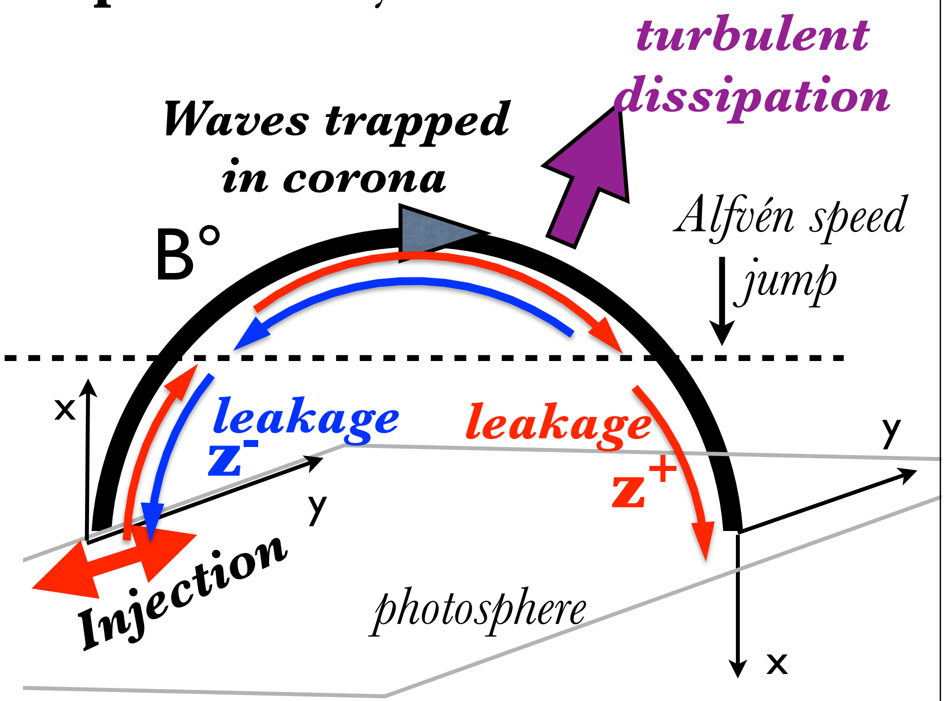
Average spectra $E^{\pm}(x,k_n)$



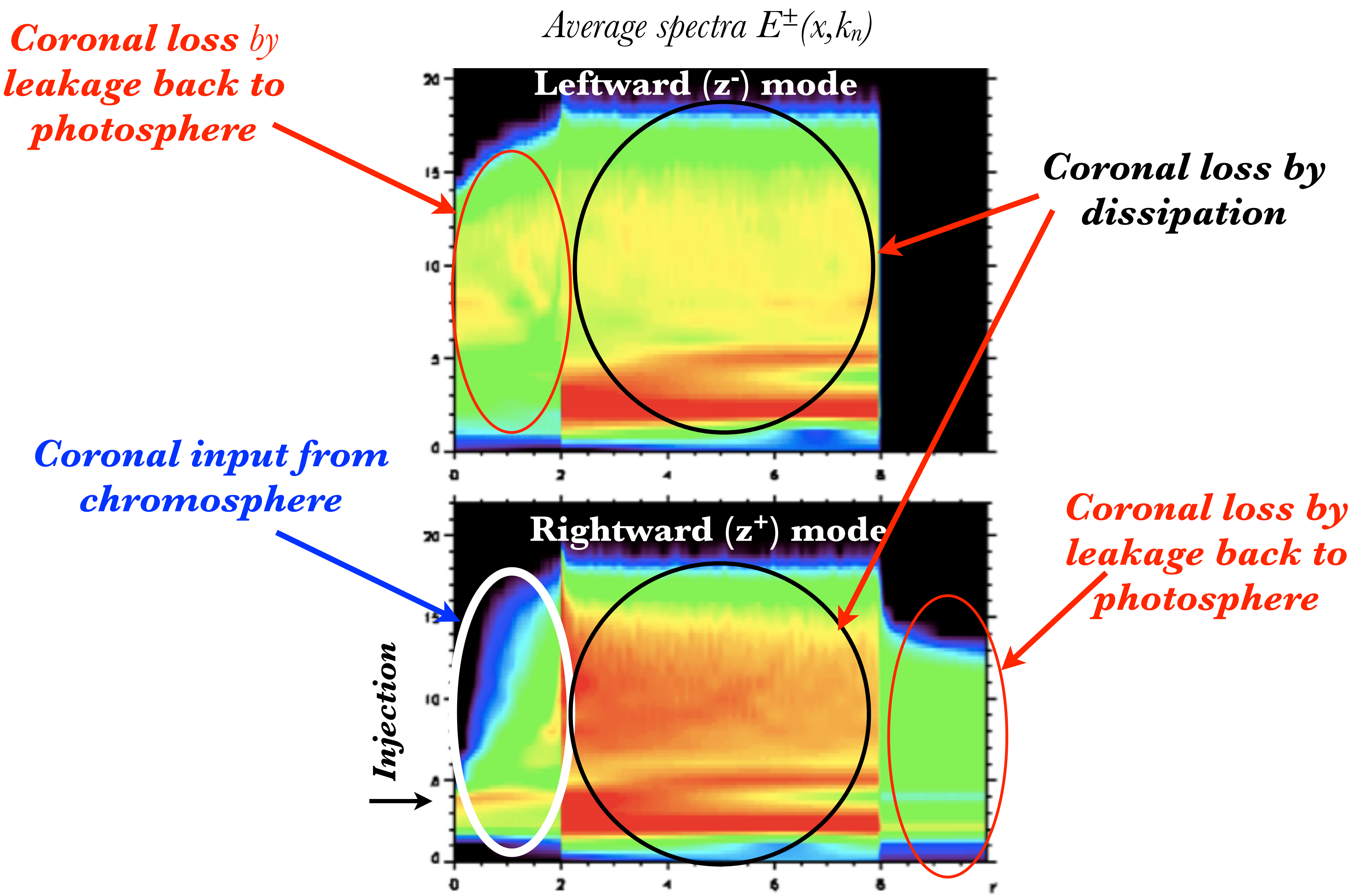
•Include **within the domain a *finite* *Alfvén speed jump***, use Shell-Atm model, and use *open* boundaries at x=0 and L (solar surface)

•Inject waves at *left* boundary (i.e., left loop foot point) x=0.

Here: **time-averaged** spectra **compensated** by $k^{5/3}$

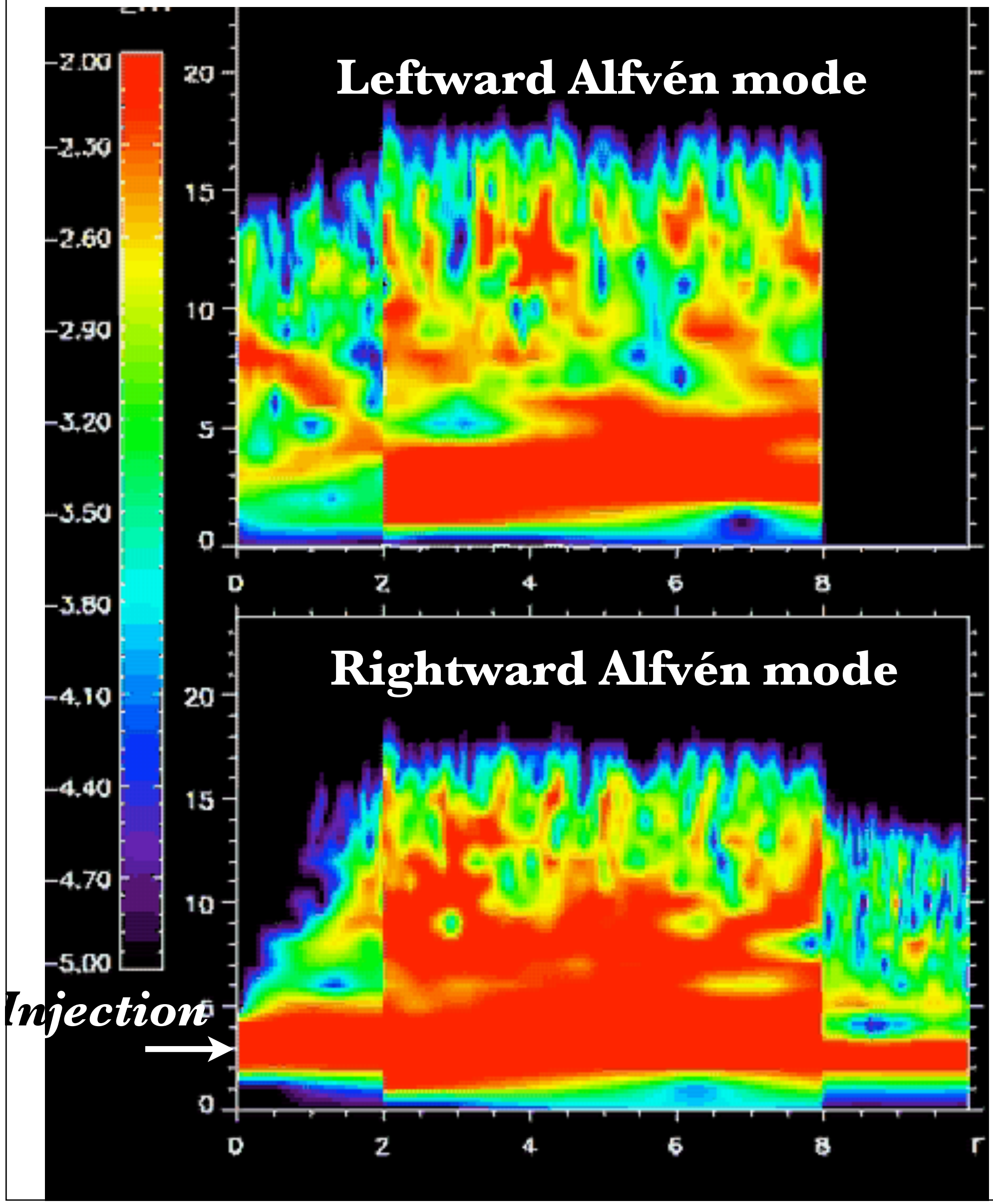


Leakage AND turbulence
SHELL model with chromosphere+corona



Leakage AND turbulence
SHELL model with chromosphere+corona

Instantaneous spectra $E^{\pm}(x,k_n)$



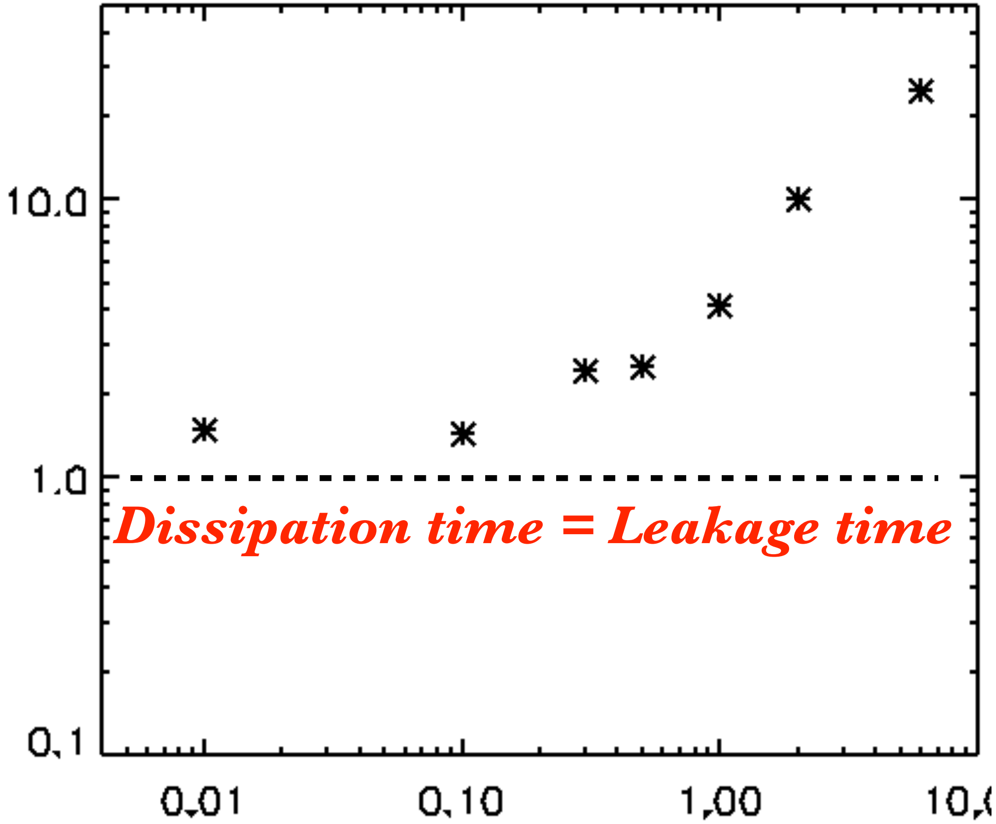
•**Include within the domain a *finite Alfvén speed jump***, use Shell-Atm model, and use *open* boundaries at x=0 and L (solar surface)

•Inject waves at *left* boundary (i.e., left loop foot point) x=0.

Spectra are compensated by $k^{5/3}$

Turbulence remains weak in corona

Coronal Dissipation time normalized by leakage time



Injection scale L_{phot}

•Fix the Alfvén coronal time and leakage time, and vary the photospheric injection scale
Measure time- and space-averaged (in corona) dissipation time
$1/t_{dis} = \langle \nu \, \Sigma k^2 E(k) dk \rangle / \langle \Sigma k^2 E(k) dk \rangle$

1. With ***large phot. injection scale***, cascade has no time to develop before energy leaks out of corona: $\mathbf{t_{dis}}$ remains large
2. ***Decreasing injection scale*** leads to stronger coronal cascade: $\mathbf{t_{dis}}$ decreases linearly
3. ***Decreasing further*** injection scale ***cannot*** force $\mathbf{t_{dis}} < \mathbf{t_{leak}}$

Conclusion part 2

Shell Atm model indicates that leakage always plays a role in turbulent dissipation in the coronal heating

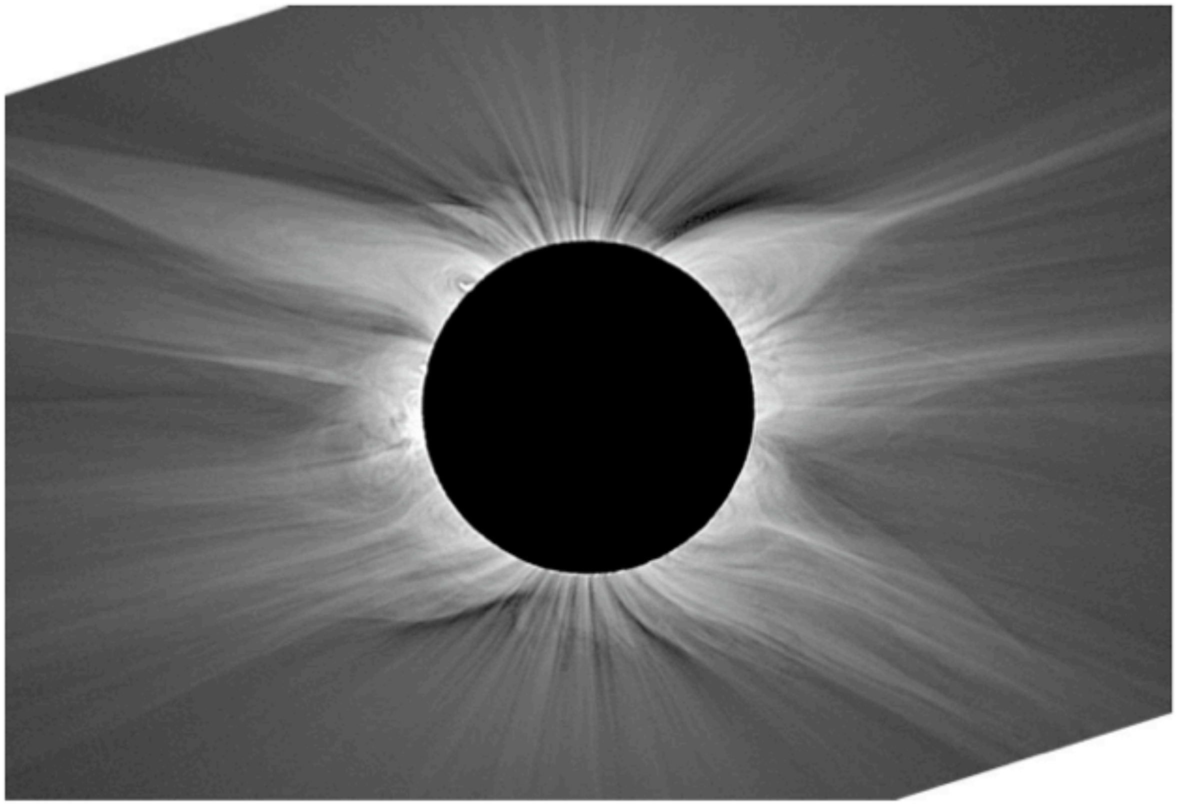
Chapter 3: Spectral anisotropy in an expanding plasma

Wind flow is very non-uniform close to the Sun, as much as the magnetic structure
However, far enough from the Sun, the flow becomes simpler, close to radial

What are the basic effects of a uniform radial wind on the dynamic evolution of an advected plasma volume?

NB What follows will be deal mainly with compressible MHD

HABBAL ET AL.



se broadband white-light image of the corona taken from Greece. The image has been processed with commercial graphics software. The orig lightly for the vertical axis to match solar north (as in the previous two images), hence the cropping, which appears at the top left and bottom rig b Strikis.)

Anisotropic spectrum in solar wind?

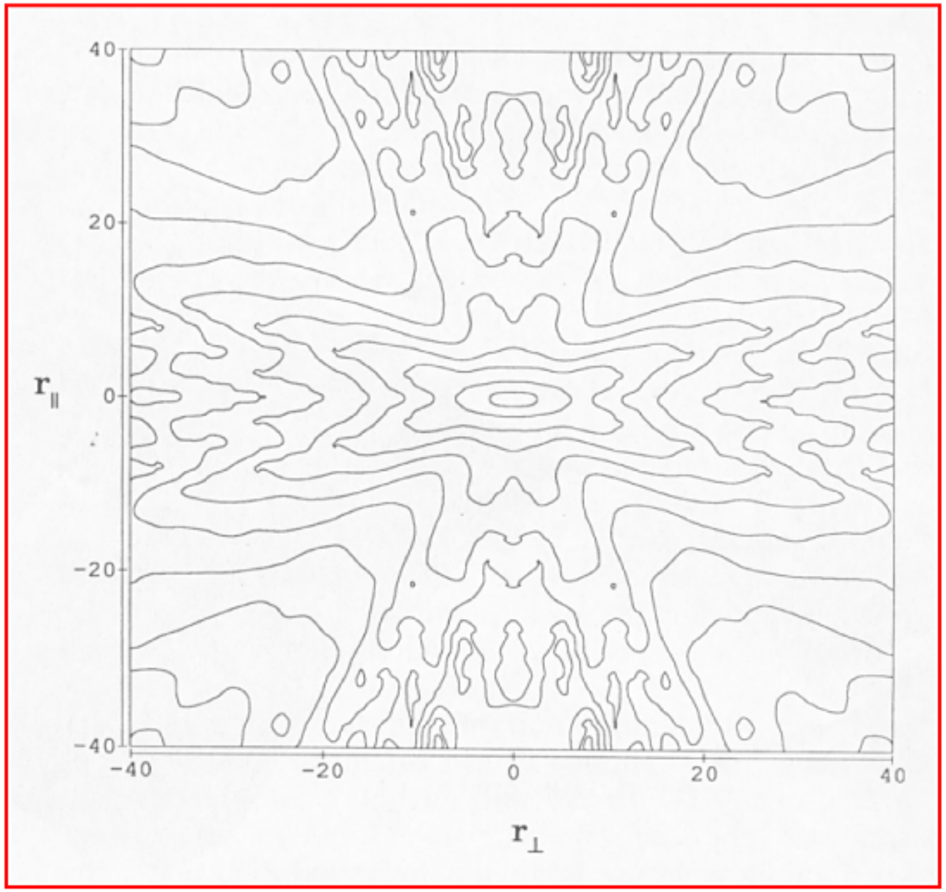
Data (u, B) are ***time-dependent*** signals.
•Taylor hypothesis ($u(r,t) = u(r-U_r t)$ can be used to recover the spatial structure *along the radial direction*

•Use fluctuations of ($\mathbf{B}°$,$\hat{e}_r$) w/time + **girotropy assumption to derive**
$\mathbf{R}(r_\perp, r_{//}) = <\delta B_i(r')\, \delta B_i(r'+r)>$

•Here: correlation function of δB as a function of $\perp$ and // distance with respect to the mean field **B**°. Units of 10^5 km $=7 10^{-4}$ AU (the *Alfvénic waveband*)

Conclusion: parallel correlation length **smaller** than perpendicular !

Correlation function $R(r_\perp, r_{//})$ of magnetic field fluctuations

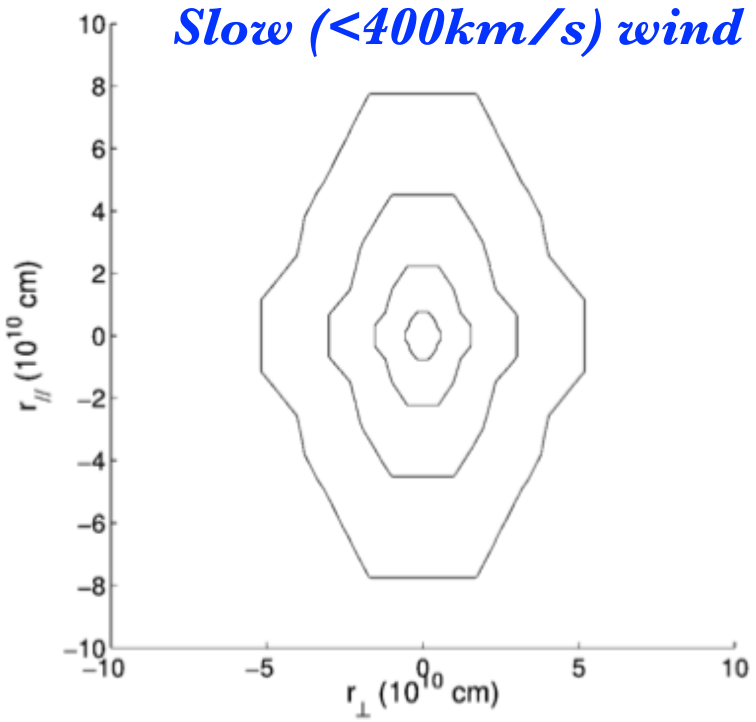


Matthaeus et al., 1990

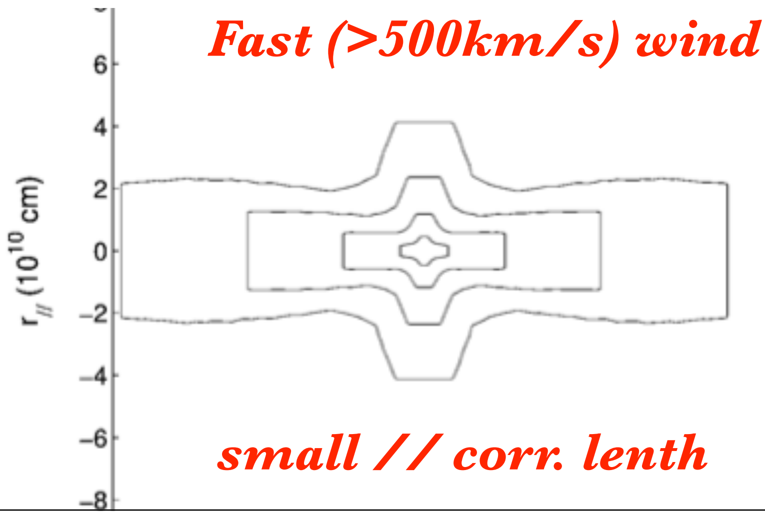
Separating fast and slow wind

Again using **gyrotropy hypothesis** around B° direction, but selecting either SLOW or FAST solar wind:

1. The **slow** solar wind has ≈ **isotropic** spectrum



2. The fast solar wind has MORE small scales in // direction: cascade is // !



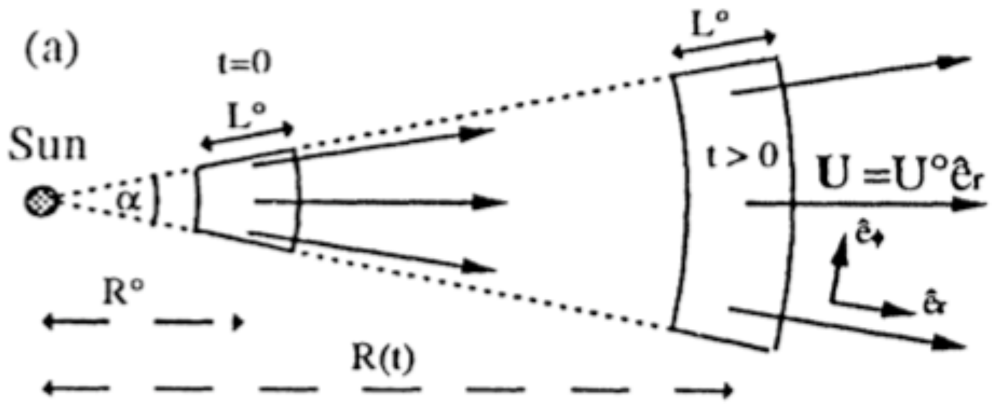
Dasso Milano Matthaeus Smith ApJ 2005

How can the wind generate a parallel component

Is there a mechanism possibly producing a spectrum with wavevectors parallel to the mean field? Yes !
The answer is: the **expansion of the wind**

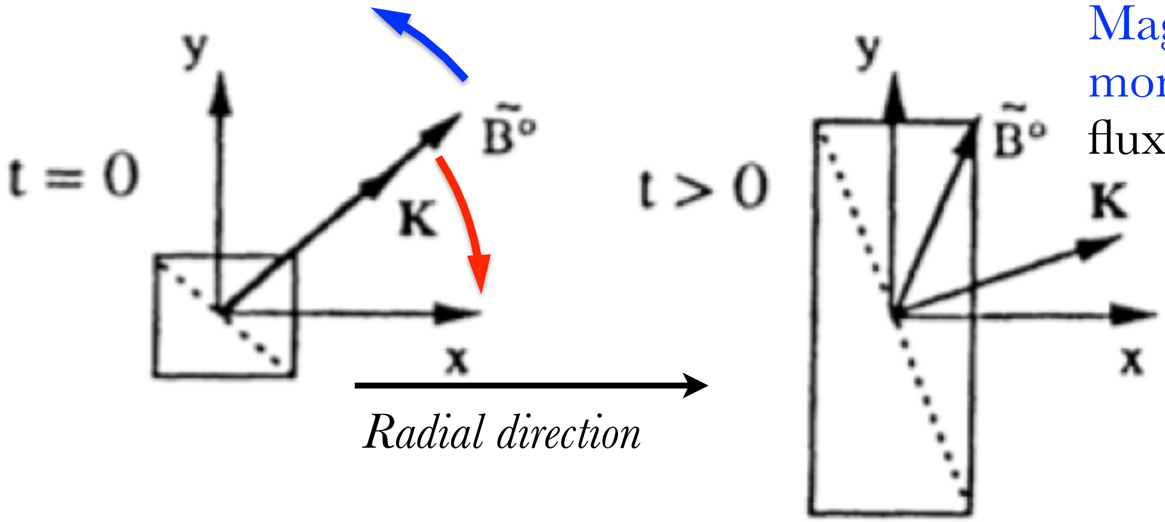
Three steps

1.Expansion is basically ⊥ to radial, which makes wave vectors cluster to the RADIAL direction
2. nonlinear coupling ⊥ to radial are reduced/delayed
3. Close enough to the Sun, magnetic field is close to radial



Numerical model of MHD/hybrid plasma <u>including expansion</u> ("expanding box model")***:*** *Grappin Velli Mangeney 1993*, *Grappin Velli 1996 (MHD), Hellinger et al 2003, 2005 (Hybrid simulations)*

Is there enough time ?



Magnetic field B° becomes more transverse because of flux conservation

Wave vectors k become more radial

Characteristic times

Time for cascade perp to B°: t_{NL}

Time for expansion: $t_{exp}=(\mathrm{div}U)^{-1} \approx R/(2U)$

=> 1 Day at 1AU, 0.1 Day at 0.1 AU

=> Expansion important ($t_{exp} < t_{NL}$) **only at large scales**

BUT Alfvénic turbulence in fast wind has **large effective t_{NL}** because z-<<z+*

=> explains why // spectrum **can dominate in fast wind**

** Dobrowolny Mangeney Veltri 1980, Grappin Frisch Pouquet Léorat 1982, Grappin Pouquet Léroat 1983, Grappin Velli Mangeney 1991*

Epilogue: evidence for the radial symmetry axis competing with the B° axis

Observational evidence of a second symmetry axis, namely the radial direction:

1. *Saur et Bieber* "Geometry of low-frequency solar wind magnetic turbulence: Evidence for radially aligned Alfénic fluctuations. JGR (1999)

and

2. *F. Sahraoui et al* 2010, 2011 (see his talk, Les Houches school)

Summary

1. Using a Shell model for reduced MHD, which allows to vary the boundary condition in the direction parallel to the mean field, we test the **critical balance hypothesis** and find it to be perfectly true in some sense, but NOT the angular spectrum which is much more anisotropic than expected.

2. Using the same model but with either semi-reflecting boundaries or a three-layers model for the solar atmosphere, we find that dissipation/heating is controlled by the competition between leakage and turbulent dissipation, **leakage being always significant**.

3. Analysis of observed solar wind turbulence shows that the **parallel component** of the magnetic spectrum in the Alfvén waveband might be **a major component**, which can be explained by the **linear effect of expansion**, which is indeed revealed by looking at the data.

All these points deserve further examination using alternate models (eg full MHD, reduced MHD).
The coronal heating paradigm (i.e. forcing by the boundaries) might be a good one to understand why turbulence properties are not universal