Spectral anisotropy of MHD turbulence with large mean field: recent numerical results

R. Grappin (1), W.-C. Müller (2), J. Léorat (1)
1: Meudon Observatory, 2: Max-Planck, Garching

Knowledge on spectral anisotropy is necessary to predict phenomena such as turbulent dissipation and heating rates.

Basic principles and phenomenologies are recalled.

The case of high-resolution (1024^3) incompressible MHD turbulence is studied.

Basic principles - incompressible limit

Incompressible => infinite β, *only Alfvén waves exist*

Notations:

density $\rho=1$; $B°$ = *mean field* = v_a = *Alfvén velocity*

- What do Alfvén waves to turbulence?

3D Navier-Stokes ($u\nabla u$) => Kolmogorov turbulence

3D MHD ($u\nabla u$ + propagation term $B°\nabla u$) => different ?

Why should Alfvén waves change something?

a) Because waves allow coherent nonlinear coupling only during short times

b) Because waves do so only when wavevector is parallel to mean field

…Assume now a **large mean field B°**

Why should Alfvén waves change turbulent cascade?

A) Because NL terms are ≈ $z^+(x,t)z^-(x,t) = f(x-v_a t)\ g(x+v_a t)$

=> Coherent interactions limited to wave travel time: $\Delta t = L/v_a$

=> Random successive interactions necessary to drive turbulent cascade -> large k

B) Because waves do so only when wavevector is parallel to mean field (in perp direction, $v_a = \mathbf{k}.\mathbf{B}° \rightarrow 0$)

=> In perp direction, turbulence proceeds as without B field

=> TWO theories of MHD turbulence :

Theory A : based on A), neglects B)

Theory B : based on B), neglects A)

Two theories of MHD (incompressible) turbulence

A) Theory A (Iroshnikov-Kraichnan, 1964, 1965)

•Random successive coherent interactions :

$\rightarrow u \approx l^{1/4}$ or $E(k) \approx k^{-3/2}$ spectrum

•Ignore anisotropy !

B) Theory B (Goldreich-Shridhar, 1995)

•Coherent interactions along perpendicular directions :

$\rightarrow u_\perp \approx l_\perp^{1/3}$ or $E(k_\perp) \approx k_\perp^{-5/3}$ spectrum (Kolmogorov)

•Neglect nonlinear interactions along B°; include ***transport*** of perp fluctuations by // waves***:***

$u\nabla u \approx B.\nabla u \approx B°.\nabla u \quad \Leftrightarrow \quad u_\perp / l_\perp \approx v_a / l_{//}$

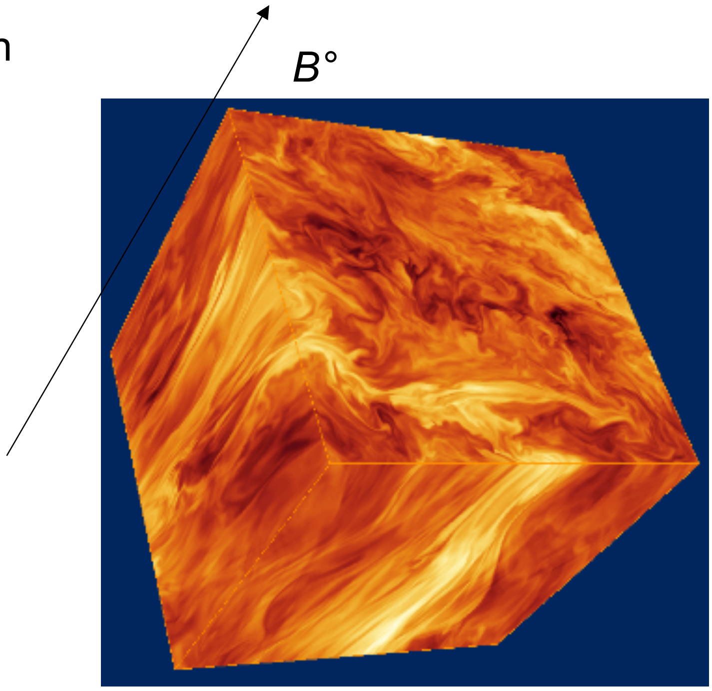
But $u_\perp \approx l_\perp^{1/3} \Rightarrow$ "critical balance":

$$k_{//} \approx k_\perp^{2/3}$$

Numerical: new (B°=5δb, 1024^3)

Anisotropy well visible in real space:

small scales are largely perpendicular to B° (Müller, 2006)



… and old (MHD 2D, B°=1, 256^2)

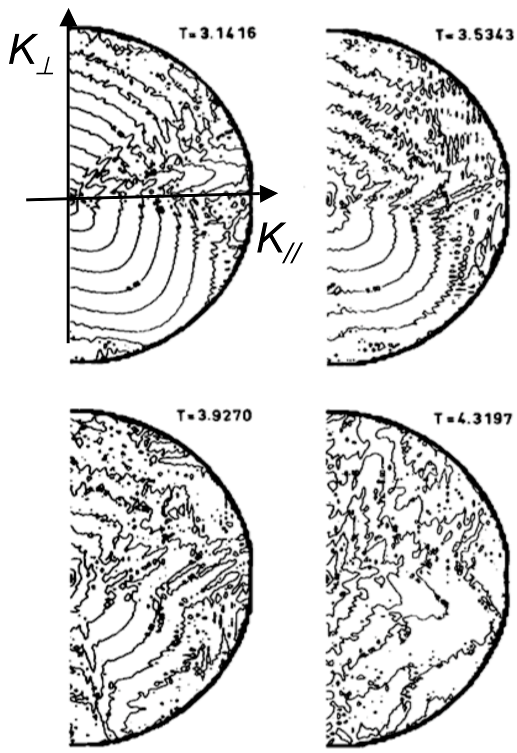
Temporal evolution of 2D energy spectrum (Grappin 1986)

NB 2D MHD with mean field is NOT turbulent (spectrum around k^{-3})

Nevertheless, one sees:

a) Early cascade towards perpendicular scales (not here)

b) Later isotropization towards parallel scales (here)



High resolution 3D: slopes depend on B°

(Müller & Grappin, 2005)

Two spectra **show definite power laws** :

(1)Total energy $E^{tot}(k)$ = kinetic + magnetic

(2)Residual energy $E^{res}(k)$ = magnetic -kinetic

Two regimes:

- B°=0 → $E^{tot} \approx k^{-5/3}$ and $E^{res}=k^{-7/3}$
- B° large → $E^{tot} \approx k_\perp^{-3/2}$ and $E^{res}=k_\perp^{-2}$

=> Single relation (valid whatever B°):

$$E^{Res}(k) = k(E^{Tot}(k))^2 \qquad (*)$$

•relation (*) results from competition between wave equipartition effect ("Kraichnan effect") and generalized dynamo effect

=> waves DO have a direct influence on turbulence ! 7

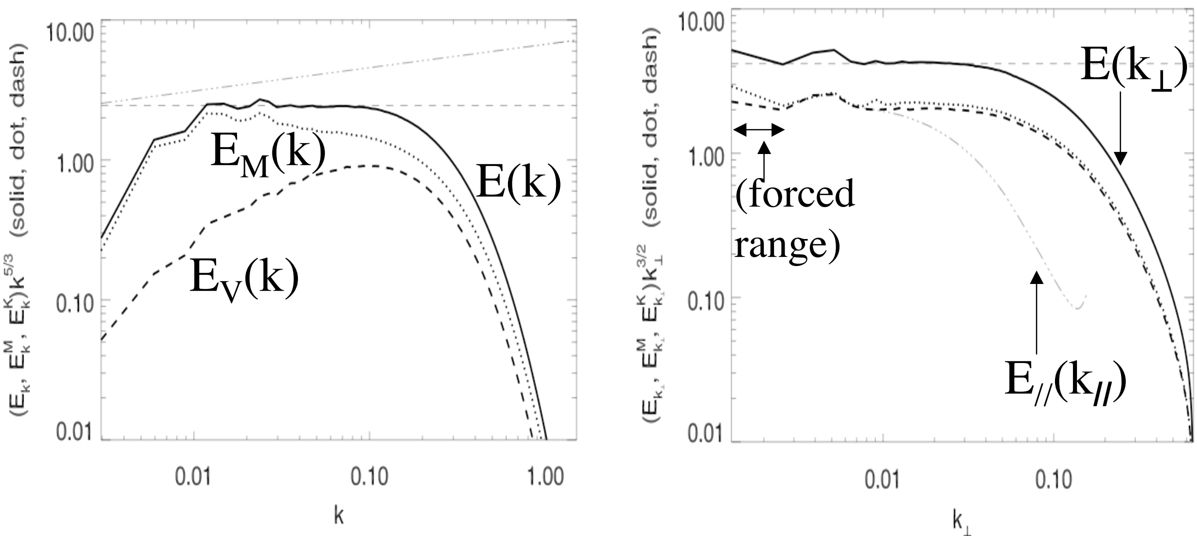
(cf. Grappin et al., 1983)

High resolution 3D: slopes depend on B°

Total energy has definite slope

(a) B°=0
decaying simulation
isotropized spectra x $\mathbf{k^{5/3}}$

(b) B°=5
forced simulation,
1D spectra x $\mathbf{k_\perp^{3/2}}$



Time-averaged spectra, wavenumbers normalized by $k_{dissipative}$

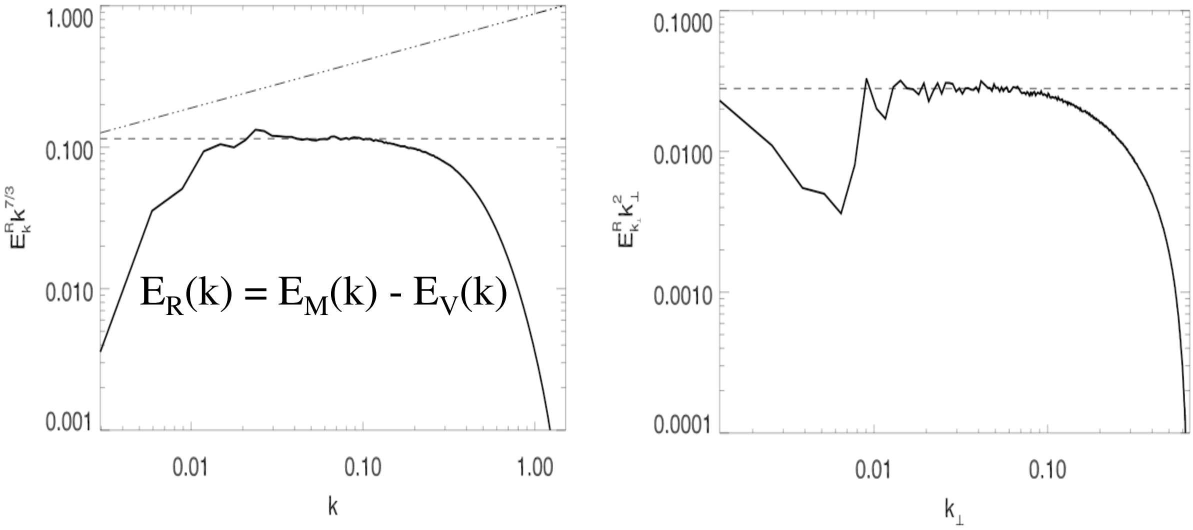
Müller Grappin PRL 95, 114502, 2005

High resolution 3D: slopes depend on B°

Residual spectrum has definite slope

(a) B°=0
decaying simulation
isotropized spectrum x $\mathbf{k^{7/3}}$

(b) B°=5
forced simulation,
1D spectrum x $\mathbf{k_\perp^2}$



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Measuring Anisotropy in 3D simulations: two methods

Consider a 3D simulation with strong B°

(1) Plot isocontours of the $E(k_{//},k_\perp)$ deduced from the 3D spectrum by integrating on the two perp directions.

Pick $(k_{//},k_\perp)$ pairs with given energy => plot relation $k_{//}(k_\perp)$

Fit with the ***anisotropy INDEX q*** :

$$k_{//} \approx k_\perp^q$$

q=1 means isotropy; q=2/3 is the GS prediction

(2) Do same, but use $(k_{//},k_\perp)$ pairs deduced from ***1D spectra***, defined NOT as in (1) from $E(0,k_\perp)$ and $E(k_{//},0)$, but as *integrals*:

$E(k_\perp) = \int dk_{//}\, E(k_{//},k_\perp)$

$E(k_{//}) = \int dk_{//}\, E(k_{//},k_\perp)$

High resolution 3D (B°=5): anisotropy index

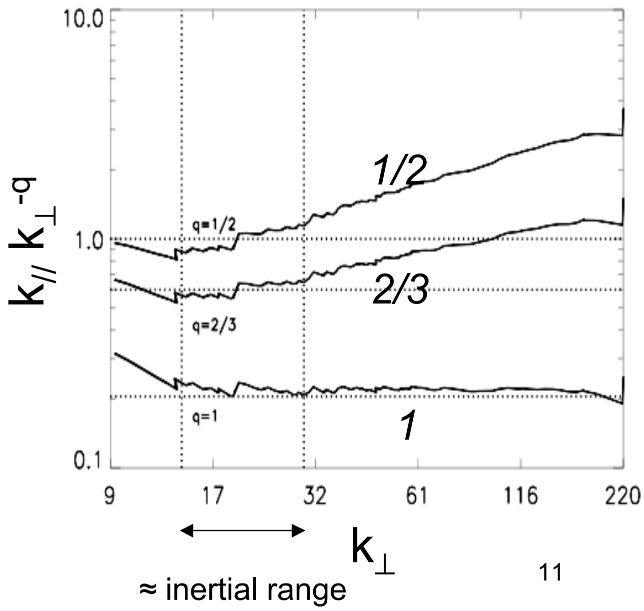
$k_{//} \, k_\perp^{-q}$ vs $k_\perp$

Anisotropy index q using method 1:

⇒ Except for the largest scales, all other scales are quasi-isotropic ($q \approx 1$)

⇒ restricted inertial range has

$1 < q < 2/3$



High resolution 3D (B°=5): anisotropy index

$k_{//} \, k_\perp^{-q}$ vs $k_\perp$

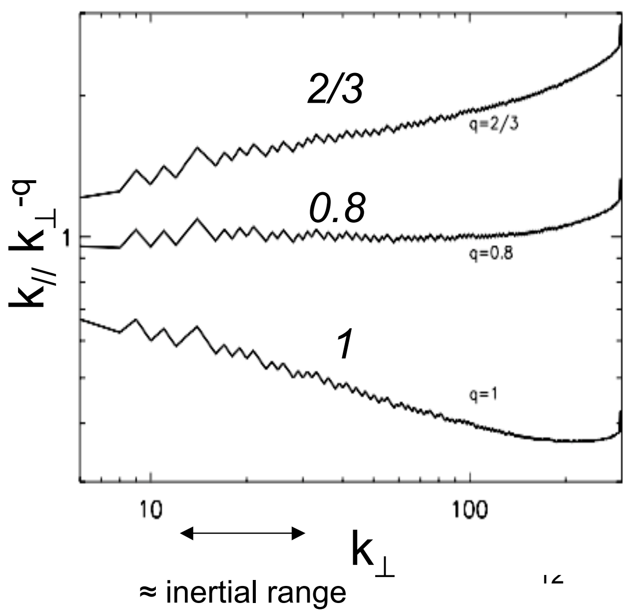
Anisotropy index q using method 2:

⇒ All scales have:

$q=0.8$

except for very small scales which are ≈ isotropic :

$q \approx 1$



Discussion

Method 2 → q=0.8 on k-inverval much larger than inertial range

Several authors find q=2/3 ("critical balance") (eg Cho et al 2003, Maron and Goldreich 2001).

Difference may be due to our larger resolution, or to method, or to different turbulence regimes.

In conclusion:

(1) our previous results on case B°=5 on spectral slopes:

$k^{-3/2}$ total energy, k^{-2} residual energy

(2) our present finding q=0.8

(1)+(2) support IK phenomenology, i.e., *some nonlinear interactions occur // B°*

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