Weak and Strong turbulence in MHD

R. Grappin (Luth, Lpp)
coll. A. Verdini (SIDC Bruxelles)

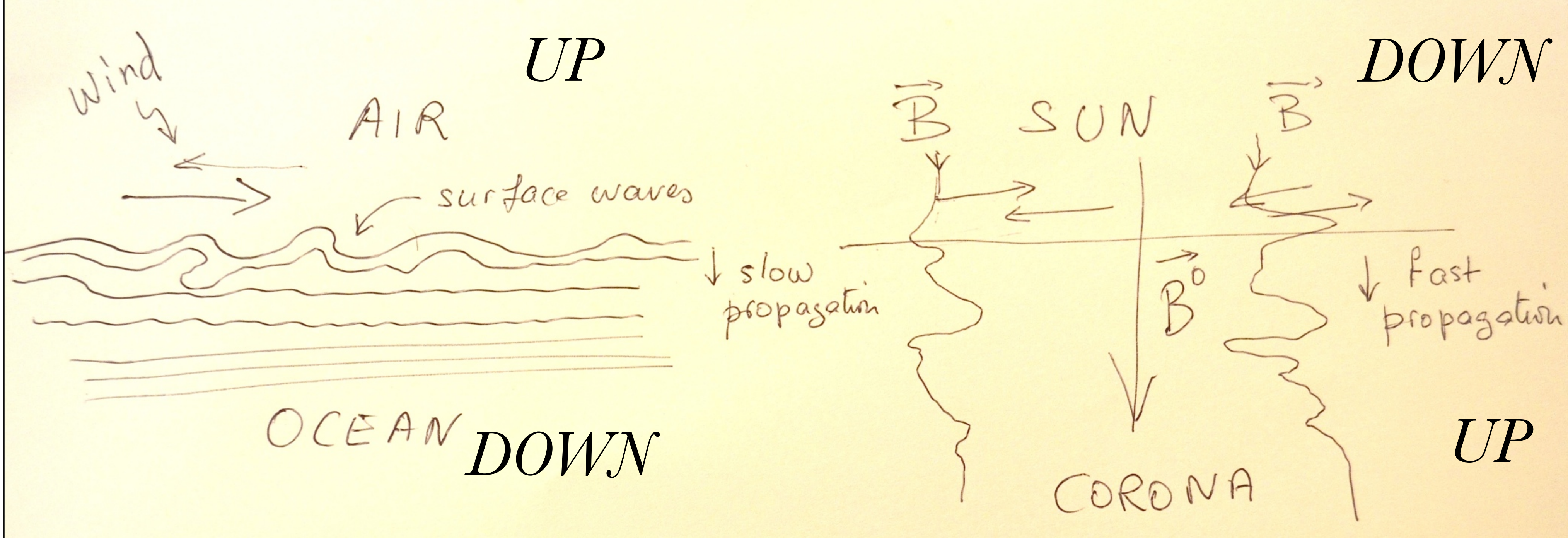
Aim: find numerical proof of weak and strong regimes for strongly anisotropic turbulence, assumed to be valid for strong mean field B°

Motivation: when large scales are forced weakly (with $k_{//}B° > k_{\perp}u$), one should find a weak regime ($k_{\perp}^{-2}$), followed by a strong regime ($k_{\perp}^{-5/3}$) at smaller scales. No numerical proof of that exists yet.
Published works report that varying the strength of large scale forcing leads to a continuous variation of the slope, but nothing like the above double scaling. (eg *Perez & Boldyrev 2008*)

The shell-Reduced MHD model (Buchlin, 2004 thesis) offers the opportunity to test these ideas as it is able to describe Reduced MHD turbulence with Reynolds $\approx 10^6$

MHD with mean field B°
allows fast transmission of surface forcing

Slow tranmission (no B°) *Fast transmission (B°≠0)*



Physical/Numerical model

1. Reduced MHD ($b/B° \ll 1$)
•No parallel gradients except linear (propagation) along B°
•No parallel polarizations ($b_{//}=u_{//}=0$)
•Incompressible limit ($\nabla_\perp . b_\perp = 0$)
=> **Quasi 2D** ($\perp$ B°), but ***not completely***: u,b =u(x), b(x)

2. "Shell" Reduced MHD (*Buchlin, thesis 2004, Nigro et al 2004, Buchlin Velli 2007*)
•solves for $\hat{\mathbf{u}}(\mathbf{x},\mathbf{k_y},\mathbf{k_z})$ (1/2 FFT)
•use scalar (wavenumber) to represent $\perp$ Fourier space by shells: $k_n \approx 2^n k°_\perp$
•allows to reach $Re \approx 10^6$

Forcing strong or weak regimes

Forcing "strong" or "weak" → cascade strong or weak

(1)"Strong" means that z+ and z- wavepackets have time to couple:

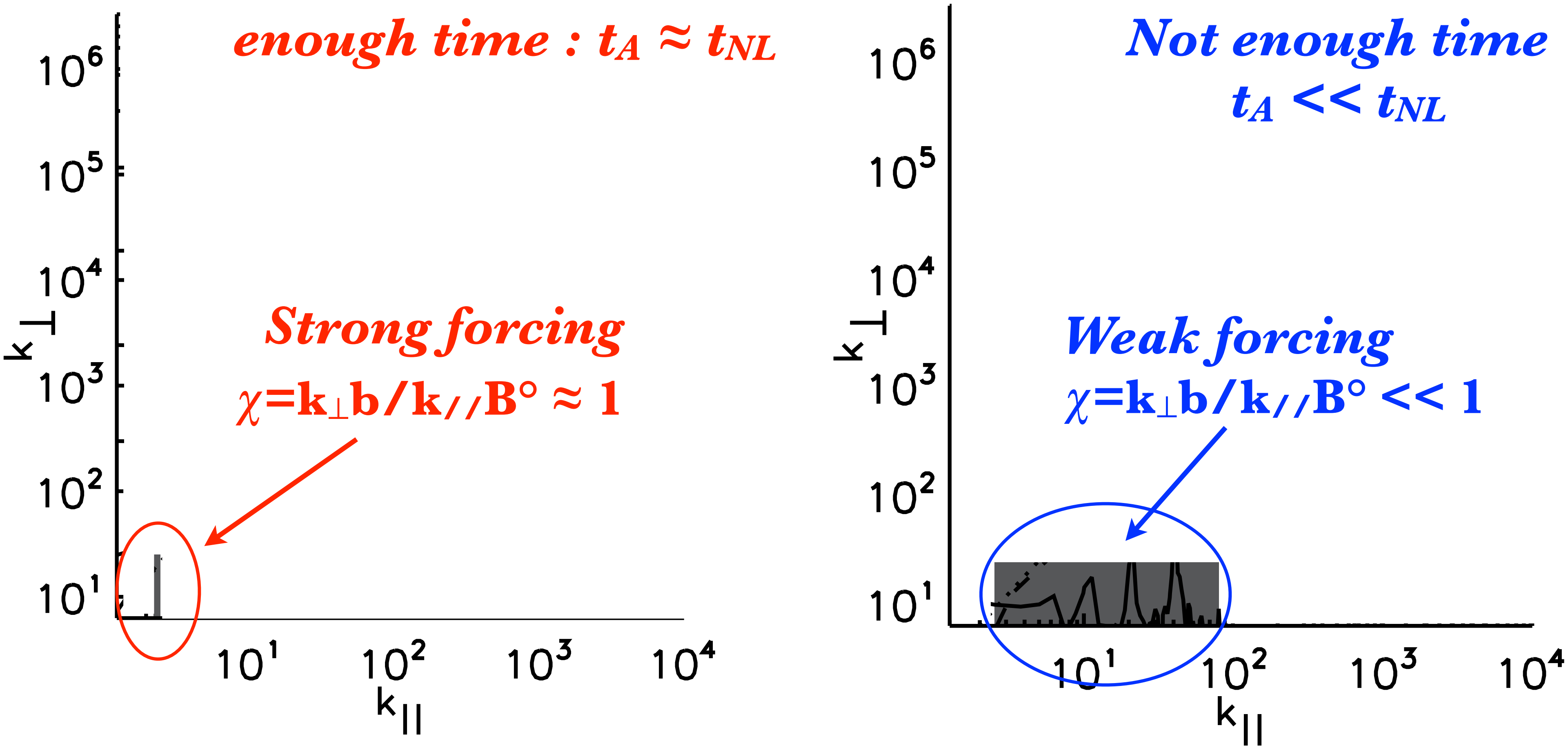
$$t_A \approx t_{NL}$$

(2)"Weak" means wavepackets pass too fast to interact completely

$$t_A << t_{NL}$$

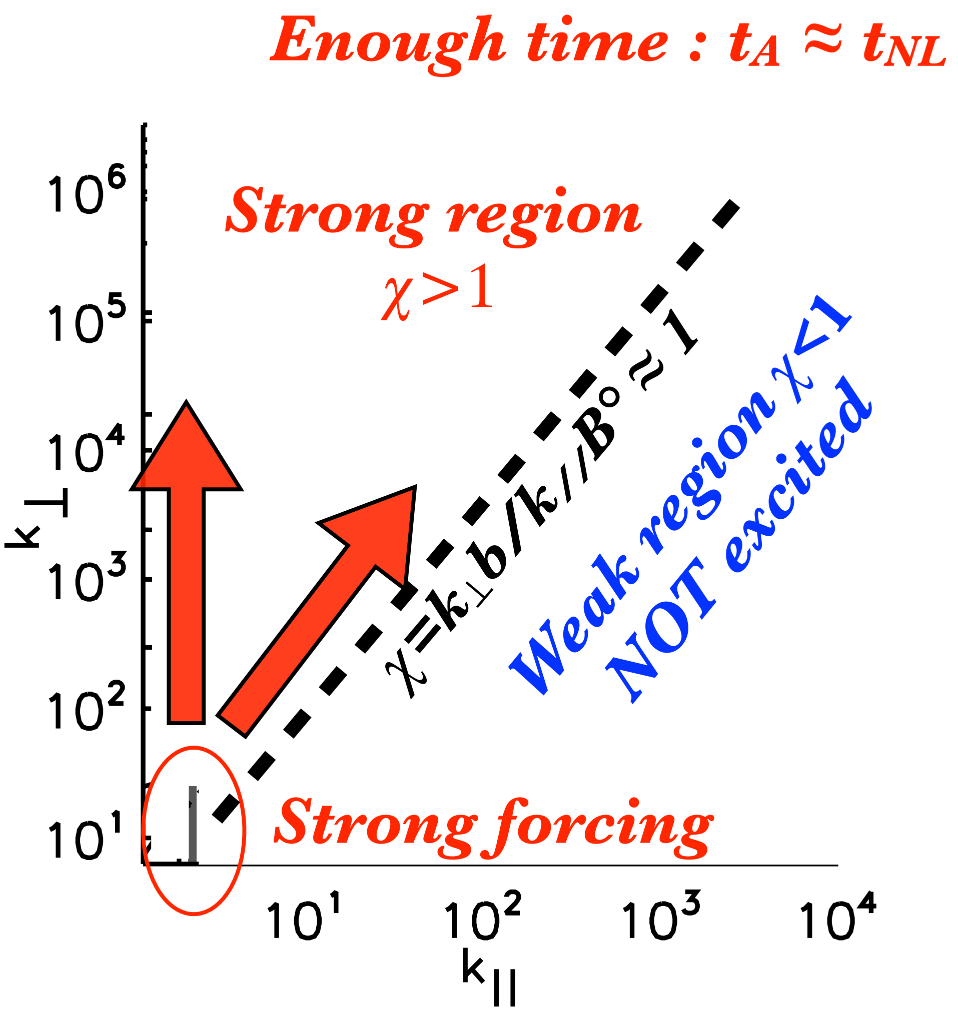
Depends on ratio of χ**=tA/tNL** = $kz^{\pm}/k_{//}B°$ at **forcing scales**

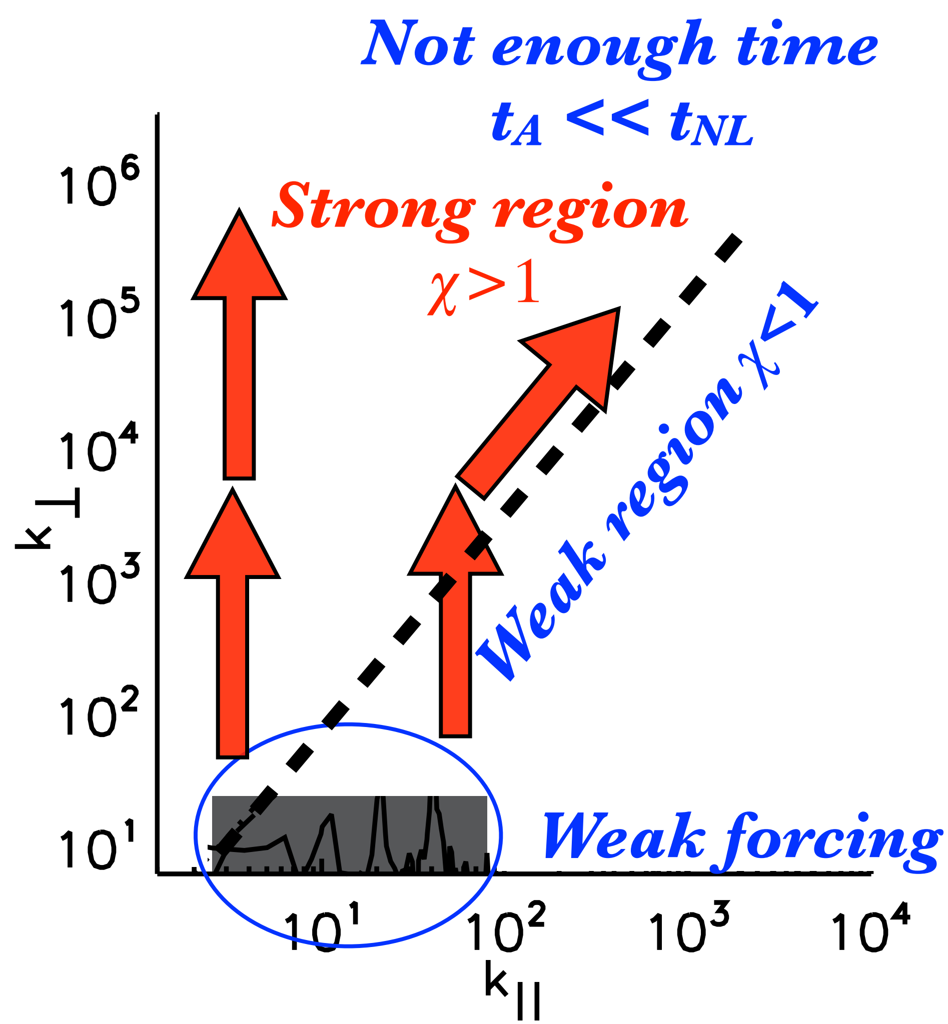
Strong and weak forcing



- In both cases $b_{rms}/B° \approx 1/5$

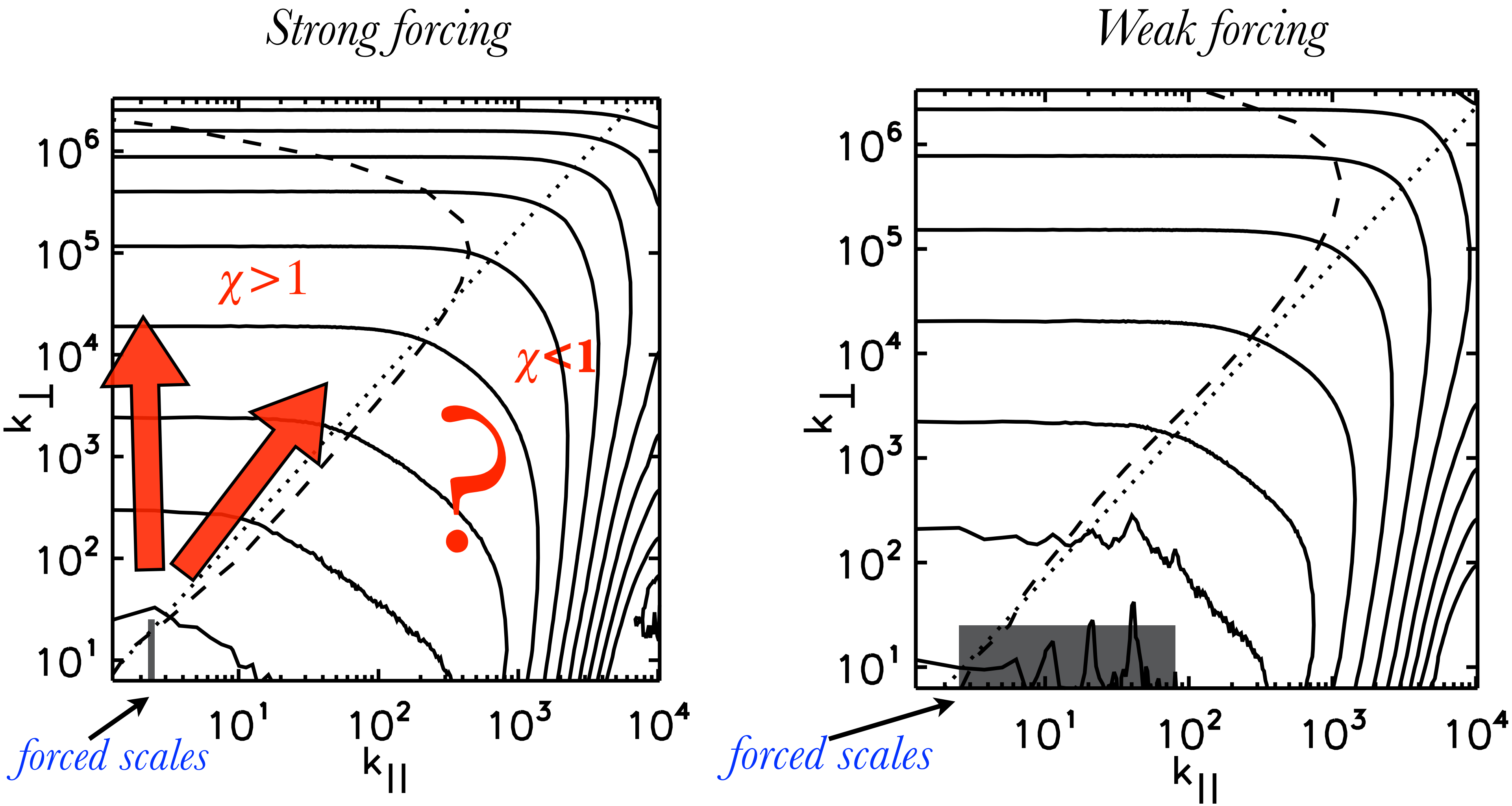
Strong and Weak: Common Wisdom





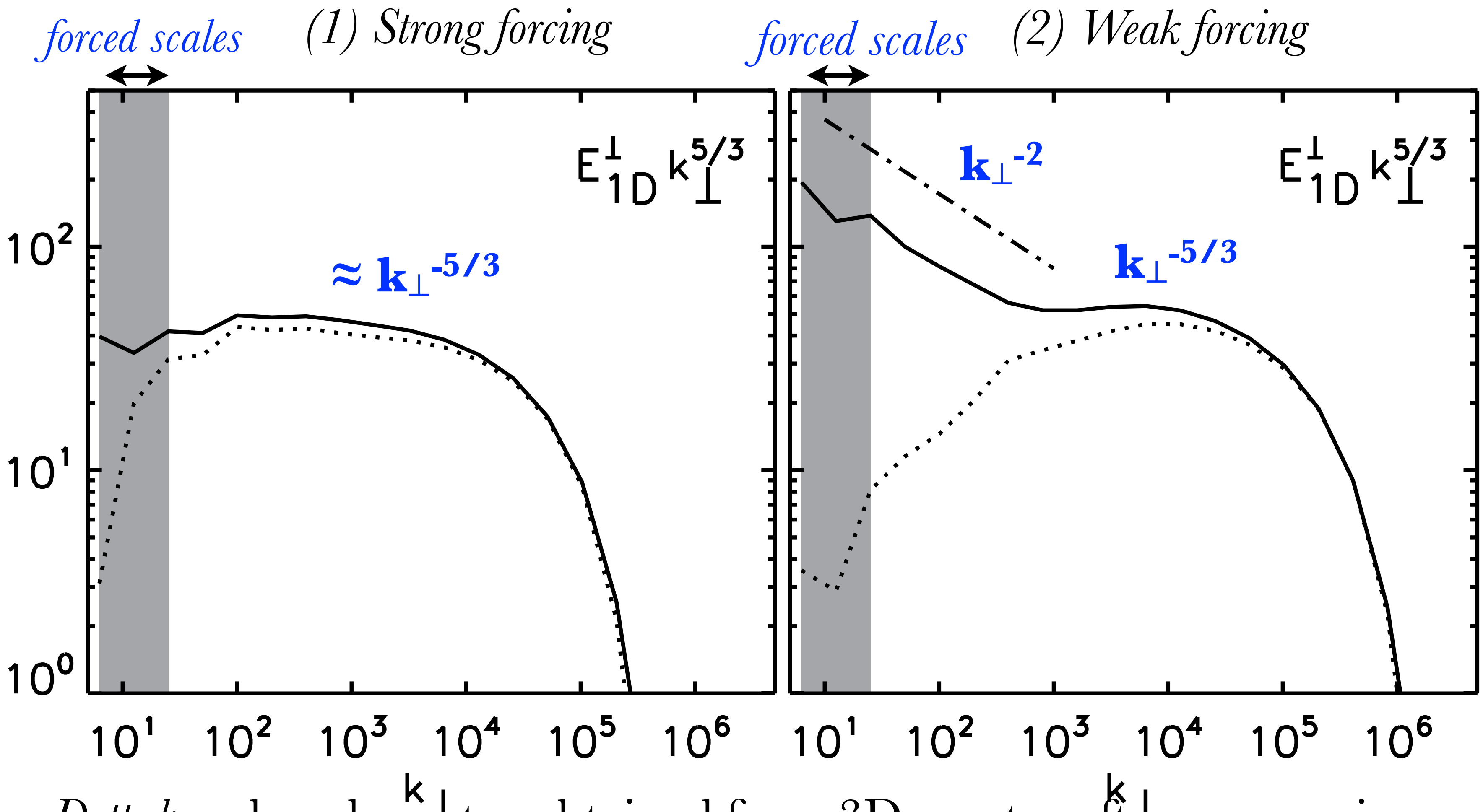
- In both cases $b_{rms}/B^\circ \approx 1/5$

Strong and weak forcing: true 3D spectra



Dashed : $\chi = 1$ isocontour; *dotted*: theoretical $\chi = 1$ isocontour (neglecting dissipation)

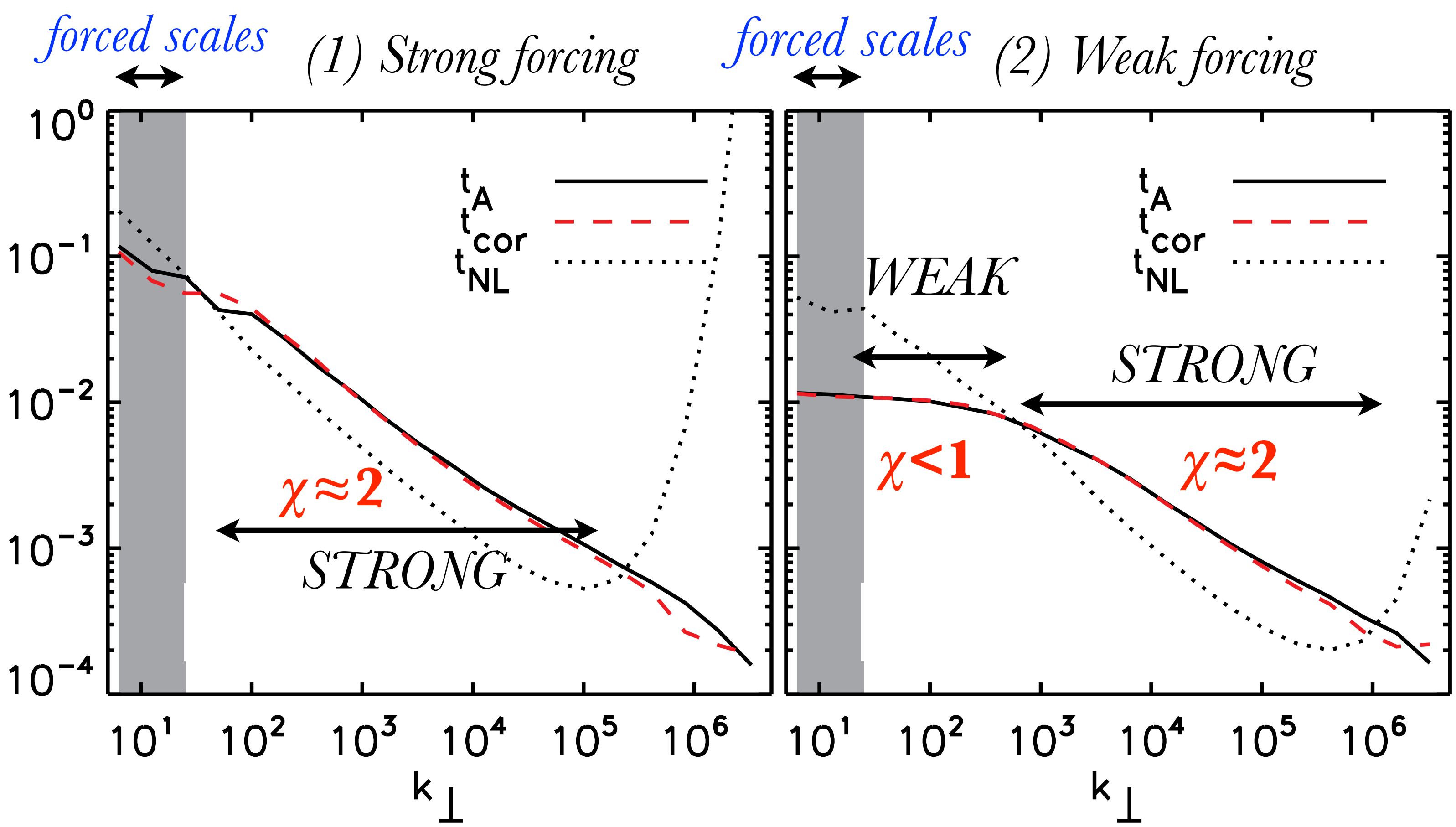
Reduced ⊥ energy spectrum $E(k_\perp)$



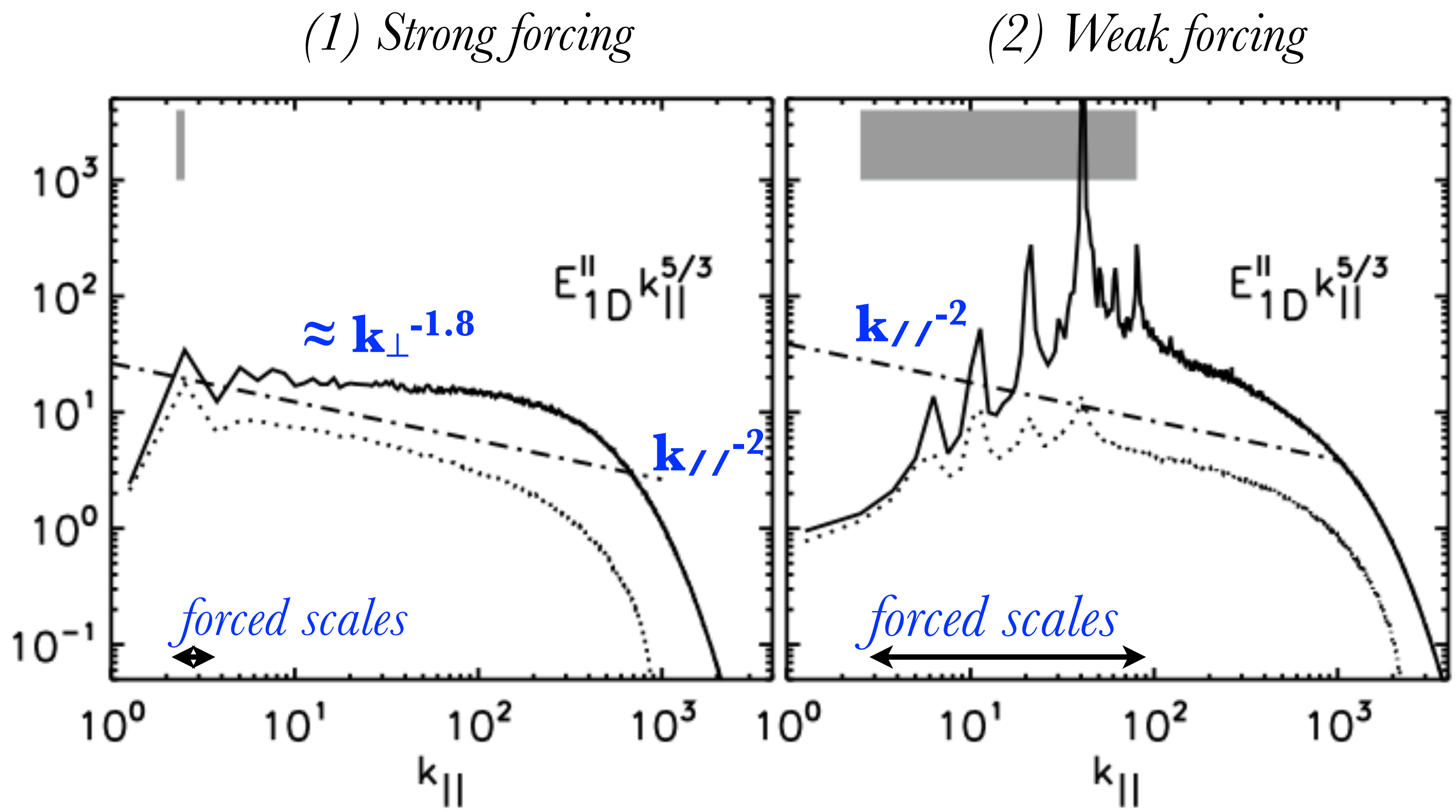
Dotted: reduced spectra obtained from 3D spectra after suppressing excitation in region $\chi<1/2$

⇒ this destroys the large scale $k_\perp^{-2}$ in the weak case (right)

Measuring time ratio $\chi = t_A / t_{NL} = 1$



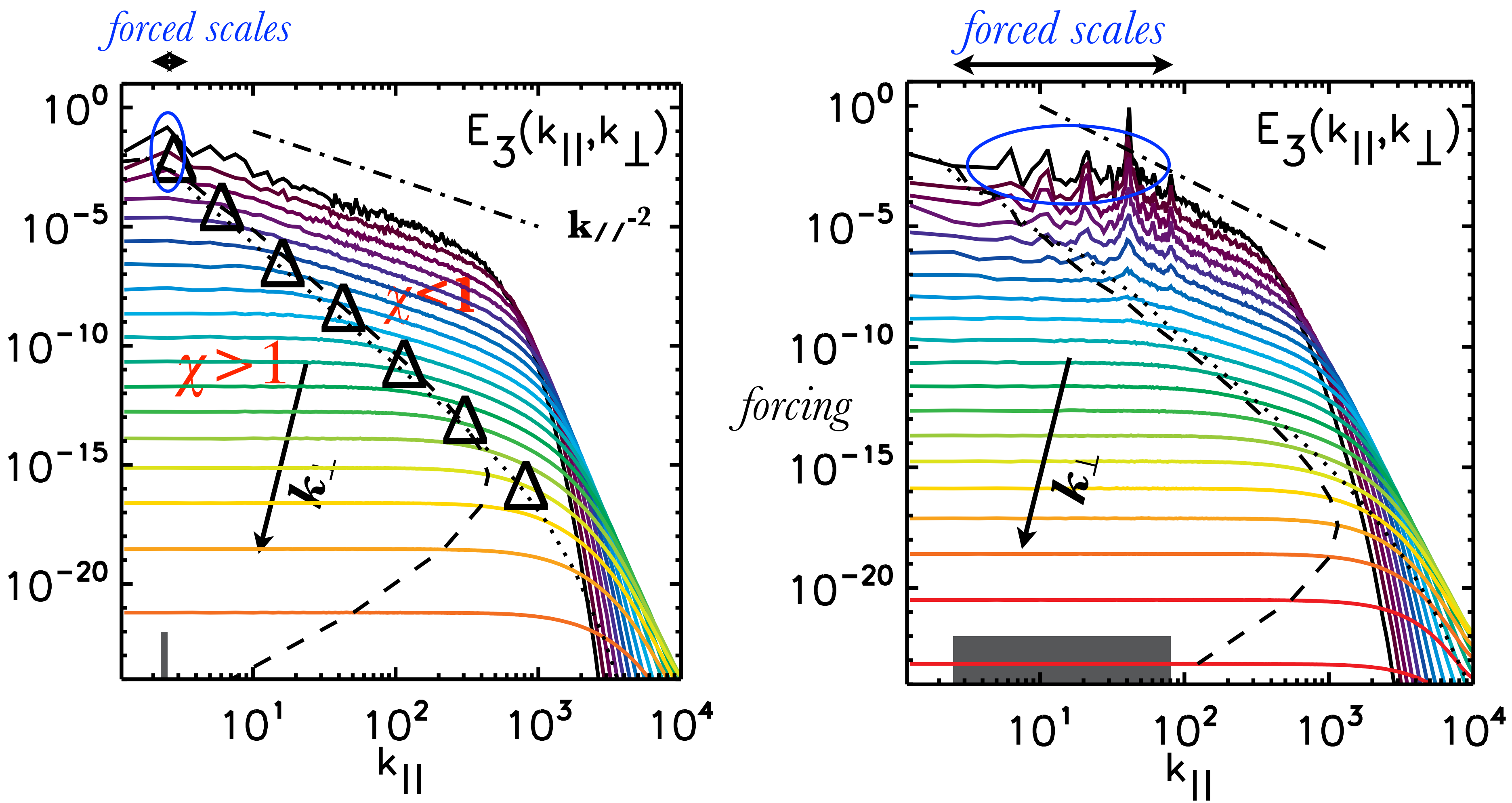
Reduced // scaling $E(k_{//})$



Left: 1D reduced parallel slope is 1.8, NOT -2: this is due to **χ<1** excitation
Dotted: reduced spectra obtained from 3D spectra after suppressing excitation in region χ<1/2

parallel scaling again

(1) Strong forcing *(2) Weak forcing*



(b) Energy in $\chi<1$ region : $\mathbf{E_3}(k_\perp,k_{//}) \propto \mathbf{k_{//}^{-2}}$

- due to $1/f^2$ spectrum of large $\perp$ eddies
- $\neq$ *reduced spectrum* $\mathbf{E_{1D}}(k_{//}) \propto \mathbf{k_{//}^{-2}}$

- (Δ) *marks boundary* of "strong" regime $\Delta f = 1/t_{cor}(k_\perp) = 1/t_{NL} = kz^\pm$

Conclusion

•We obtained here for the first time the double scaling cascade ($k_\perp^{-2}$ at large scales then $k_\perp^{-5/3}$) of the usual anisotropic phenomenology of RMHD
This result was never obtained in MHD/RMHD perhaps due to too small Reynolds numbers.
•We precisely measured the relevant time scales and found good agreement with both critical balance and weak theory too.
•Interestingly, parallel small scales do get excited in the weak region with a definite scaling. This scaling corresponds to a f^{-2} spectrum providing a tail of harmonics to the turnover time in the strong regime.
The scaling is different when using weak forcing.
•Using the same model and adding large-scale stratification and the associated imbalance between the two Alfvén species leads to different scaling, which might be the source of the large scale 1/f spectrum observed in the solar wind (see poster by Grappin Verdini Velli).